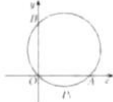


高三数学参考答案

1. D 解: 将数据从小到大排列: 2, 3, 5, 7, 9, 10, 16, 18, 20, 23. $\therefore i=10 \times 75\% = 7.5$, 则取第 8 个数, 选 D.
2. C 解: 由题意得 $8p=1$, \therefore 抛物线方程为 $x^2 = \frac{1}{4}y$, \therefore 点 $F\left(0, \frac{1}{16}\right)$, $\therefore K_{OF} = \frac{4 - \frac{1}{16}}{1 - 0} = \frac{63}{16}$, 故选 C.
3. B 解: 对于 A, 若 $m \perp \alpha, m \perp n$ 则 $n \subset \alpha$ 或 $n \parallel \alpha$
对于 B, 若 $m \parallel n, m \perp \alpha$ 则 $n \perp \alpha$
对于 C, 若 $m \parallel \alpha, m \perp n$ 则 $n \subset \alpha$ 或 $n \parallel \alpha$ 或 n 与 α 相交
对于 D, 若 $\alpha \parallel \beta, l \subset \alpha, n \subset \beta$ 则 $l \parallel n$ 或 l 与 n 异面
4. C 解: 令 $n=1$, $\frac{1}{2}S_1+1=a_1$, $a_1=2$, 由: $\frac{1}{2}S_{n-1}+1=a_{n-1} (n \geq 2, n \in \mathbb{N}^+)$, 得 $a_n = 2a_{n-1}$
 $\therefore \{a_n\}$ 是公比为 2 的等比数列, $S_5 = \frac{2(1-2^5)}{1-2} = 62$, 选 C.
5. D 解: 当 AB 过圆心 O 时, $|AB|$ 最大为 4, 当 $AB \perp OP$ 时, $|AB|$ 最小为 $2\sqrt{4-2} = 2\sqrt{2}$,
 $\therefore |AB| \in [2\sqrt{2}, 4]$, 故选 D.
6. A 解: $\because PA \cdot PB = 0$, $\therefore P$ 在以 AB 为直径的圆上, 设 AB 中点为 C,
则 P 在 P_1 处时原式取最小值, $OP \cdot OB = OP \cdot OB = (1-\sqrt{2}) \times 2 = 2-2\sqrt{2}$,
 \therefore 选 A.
7. C 解: $\because \frac{1}{\tan A} + \frac{1}{\tan B} = \frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} = \frac{\sin(A+B)}{\sin A \sin B} = \frac{\sin C}{\sin A \sin B} = 2$
 $\therefore \frac{\sin^2 C}{\sin A \sin B} = 2, \frac{\sqrt{3}}{2} = \sqrt{3} = \frac{c^2}{ab}, \therefore ab = \sqrt{3}$, 选 C.
8. A 解: 过 M 作 $MB \perp x$ 轴, 由题意可得 $\triangle OAD \sim \triangle OMB$, $\frac{OD}{OA} = \frac{OB}{OM}$, $\therefore |OD| \cdot |OM| = |OA| \cdot |OB|$
设点 $M(x_0, y_0)$, 直线 AM 的斜率为 k,
则直线 AM 方程: $y - y_0 = k(x - x_0)$ 与双曲线 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 联立,
可得 AM 方程为 $\frac{x-x_0}{a^2} - \frac{y-y_0}{b^2} = 1$, $\therefore A\left(\frac{a^2}{x_0}, 0\right), B(x_0, 0)$, $\therefore |OA| \cdot |OB| = a^2$.
9. ACD 解: $\because \sin\left(2x - \frac{\pi}{6}\right) \in [-1, 1]$, $\therefore f(x) \in [1, 5]$, \therefore A 正确
令 $2x - \frac{\pi}{6} = k\pi$, 则 $x = \frac{\pi}{12} + \frac{k\pi}{2}$, $\therefore f(x)$ 的对称中心为 $\left(\frac{\pi}{12} + \frac{k\pi}{2}, 3\right), k \in \mathbb{Z}$, \therefore B 错误
令 $-\frac{\pi}{2} + 2k\pi \leq 2x - \frac{\pi}{6} \leq \frac{\pi}{2} + 2k\pi$, 解得 $-\frac{\pi}{6} + k\pi \leq x \leq \frac{\pi}{3} + k\pi$, 令 $k=0$, 得 $x \in \left[0, \frac{\pi}{3}\right]$.
 \therefore C 正确 令 $2x - \frac{\pi}{6} = \frac{\pi}{2} + k\pi$, 解得 $x = \frac{\pi}{3} + \frac{k\pi}{2}$, $\therefore f(x)$ 在 $\left(0, \frac{5}{6}\pi\right)$ 上的唯一极值点为 $\frac{\pi}{3}$.
 \therefore D 正确, 故选 ACD.



10. BC 解: $n(M) = n(N) = A_1^1 + C_2^1 A_2^1 = 12$, $n(MN) = C_2^2 + C_1^1 C_1^1 = 5$, $n(\Omega) = C_3^1 A_1^1 = 36$

$$\therefore P(M) = P(N) = \frac{12}{36} = \frac{1}{3}, P(MN) = \frac{5}{36}, \therefore B \text{ 正确}$$

$\because M$ 与 N 可能同时发生, $\therefore M$ 与 N 不互斥, $\therefore A$ 错误

$$\therefore P(\overline{MN}) = 1 - P(MN) = 1 - \frac{5}{36} = \frac{31}{36}, \therefore C \text{ 正确}$$

$$\therefore P(M \cup N) = P(M) + P(N) - P(MN) = \frac{1}{3} + \frac{1}{3} - \frac{5}{36} = \frac{19}{36}, \therefore D \text{ 错误}$$

故选 BC.

11. ACD 解: 对于 A, 令 $x=y=0$ 得 $f(0+0) = f(0) + f(0) + 0$, $\therefore f(0) = 0$, 故 A 正确

对于 B, 令 $x=y=1$ 得 $f(2) = f(1) + f(1) + 1 = 1$,

$$\text{令 } x=1, y=-1 \text{ 得 } f(0) = f(1) + f(-1) - 1, \therefore f(-1) = 1 - f(2), \text{ 故 B 错误}$$

对于 C, 令 $x=n, y=1$ 得 $f(n+1) = f(n) + f(1) + n$, $\therefore f(n+1) - f(n) = n$

$$\therefore f(2024) = f(1) + f(2) - f(1) + f(3) - f(2) + \dots + f(2024) - f(2023) \\ = 0 + 1 + 2 + 3 + \dots + 2023 = 1012 \times 2023, \text{ 故 C 正确}$$

对于 D, 令 $y=1$ 得 $f(x+1) = f(x) + x$, 两边同时求导得 $f'(x+1) = f'(x) + 1$,

$$\therefore f'(k+1) - f'(k) = 1, \therefore \sum_{k=1}^{2024} f'(k) = 2024f'(1) + \frac{2024 \times 2023}{2} = 1012 \times 2024,$$

故 D 正确

12. [3] 解: 易知 $A = \{1, 2, 3\}$, $B = (2, +\infty)$, $\therefore A \cap B = \{3\}$

13. $\frac{\sqrt{2}}{4}$ 解: 设 A, C 中点为 O, O 到平面 DCE 距离为 A_1 到平面 DCE 距离的一半, 设 A_1 到平面

CDE 的距离为 d, 由 $V_{A_1-CDE} = V_{C-A_1DE}$, 则 $\frac{1}{3}S_{\triangle CDE} \cdot d = \frac{1}{3}S_{\triangle CA_1E} \cdot |CD|$

$$\therefore d = \frac{\frac{1}{2} \times |x| \times 1}{\frac{1}{2} \times 1 \times \sqrt{2}} = \frac{\sqrt{2}}{2}, \therefore O \text{ 到平面 } CDE \text{ 的距离为 } \frac{\sqrt{2}}{4}$$

14. 1 $\left[1, \frac{4\sqrt{2}+5}{7}\right]$ 解: (1) 由题 $a^2 - ab + b^2 = 1$, 得 $a^2 - ab + \frac{1}{4}b^2 + \frac{3}{4}b^2 = 1$, 得 $\left(a - \frac{1}{2}b\right)^2 + \left(\frac{\sqrt{3}}{2}b\right)^2 = 1$

$$\text{令 } \begin{cases} a - \frac{1}{2}b = \cos \theta \\ \frac{\sqrt{3}}{2}b = \sin \theta \end{cases}, \theta \in [0, 2\pi], \text{ 得 } \begin{cases} a = \frac{\sqrt{3}}{3} \sin \theta + \cos \theta \\ b = \frac{2\sqrt{3}}{3} \sin \theta \end{cases}$$

$$\therefore ab = \left(\frac{\sqrt{3}}{3} \sin \theta + \cos \theta\right) \frac{2\sqrt{3}}{3} \sin \theta = \frac{2}{3} \sin^2 \theta + \frac{2\sqrt{3}}{3} \sin \theta \cdot \cos \theta = \frac{2}{3} \sin\left(2\theta - \frac{\pi}{6}\right) + \frac{1}{3},$$

$$\therefore ab \in \left[-\frac{1}{3}, 1\right], ab \text{ 的最大值为 } 1$$



(1) 另解: 由题 $1+ab=a^2+b^2 \geq 2|a||b|=2|ab|$, 当且仅当 $|a|=|b|$ 时取等

① 当 $ab \geq 0$ 时, $1+ab \geq 2ab$, $ab \in [0, 1]$

② 当 $ab < 0$ 时, $1+ab \geq -2ab$, $ab \in [-\frac{1}{3}, 0]$, 综上: $ab \in [-\frac{1}{3}, 1]$

(2) 由题知, 原式 $= \frac{a^2+b^2+2}{(a^2+1)(b^2+1)} = \frac{3+ab}{a^2b^2+ab+2}$, 令 $t=ab \in [-\frac{1}{3}, 1]$

$$\therefore \text{原式} = \frac{t+3}{t^2+t+2} = \frac{t+3}{(t+3)^2-5(t+3)+8} = \frac{1}{t+3+\frac{8}{t+3}-5}, t+3 \in [\frac{8}{3}, 4]$$

$$\text{令 } m=t+3 \in [\frac{8}{3}, 4], m+\frac{8}{m} \in [4\sqrt{2}, 6], m+\frac{8}{m}-5 \in [4\sqrt{2}-5, 1], \therefore \text{原式} \in [1, \frac{4\sqrt{2}+5}{7}]$$

15. 解: (1) 已知数列 $\{a_n\}$ 为等差数列,

$$\begin{cases} 2S_3 = a_1 + a_4 \\ S_3^2 = (a_1-1)(S_3-1) \end{cases} \Rightarrow \begin{cases} d = 2a_1 \\ (2a_1+d)^2 = (a_1+d-1)(3a_1+3d-1) \end{cases} \Rightarrow 11a_1^2 - 12a_1 + 1 = 0$$

$$\Rightarrow a_1 = 1 \text{ 或 } \frac{1}{11} \text{ (舍)}, \therefore a_n = 2n-1, n \in \mathbb{N}^*$$

$$(2) b_n = (2n-1) + \frac{1}{(2n-1)(2n+1)} = (2n-1) + \left(\frac{1}{2n-1} - \frac{1}{2n+1}\right) \times \frac{1}{2}$$

$$\therefore T_n = b_1 + b_2 + \dots + b_n = n^2 + \frac{1}{2} \left(1 - \frac{1}{2n+1}\right) = n^2 + \frac{n}{2n+1}$$

16. 解: (1) $P_1 = \frac{C_1^2 C_2^2}{C_4^4} = \frac{2}{20}$ (1个20元, 0个10元)

$$P_2 = \frac{C_2^2 C_2^2 + C_1^2 C_3^2}{C_4^4} = \frac{4}{20} \text{ (2个20元, 0个10元或1个10元)}$$

$$\therefore P = P_1 + P_2 = \frac{6}{20} = \frac{3}{10}$$

(2) 抽取一次得分情况为随机变量 ξ , ξ 的取值为: 2, 1, 0

$$\text{抽取一次得二分: } P(\xi=2) = \frac{C_2^2 C_2^2 + C_1^2 C_3^2 + C_3^2 C_1^2}{C_4^4} = \frac{3}{10}$$

$$\text{抽取一次得一分: } P(\xi=1) = \frac{C_2^2 C_2^2 + C_1^2 C_3^2 + C_3^2 C_1^2}{C_4^4} = \frac{3}{5}$$

$$\text{抽取一次得零分: } P(\xi=0) = \frac{C_1^2 C_3^2}{C_4^4} = \frac{1}{10}$$

\therefore 抽取两次的得分情况为随机变量 X , X 的取值为: 4, 3, 2, 1, 0, 则 X 的分布列为:

$$P(X=4) = P(\xi=0) \times P(\xi=2) = \frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$$

$$P(X=3) = 2 \times P(\xi=1) \times P(\xi=3) = 2 \times \frac{3}{10} \times \frac{3}{5} = \frac{9}{25}$$

$$P(X=2) = 2 \times P(\xi=0) \times P(\xi=2) + P(\xi=1) \times P(\xi=1) = \frac{21}{50}$$

$$P(X=1) = 2 \times P(\xi=0) \times P(\xi=1) = \frac{3}{25}$$

$$P(X=0) = P(\xi=0) \times P(\xi=0) = \frac{1}{100}$$

X	4	3	2	1	0
P	$\frac{9}{100}$	$\frac{9}{25}$	$\frac{21}{50}$	$\frac{3}{25}$	$\frac{1}{100}$

$$\therefore E(X) = 4 \times \frac{9}{100} + 3 \times \frac{9}{25} + 2 \times \frac{21}{50} + 1 \times \frac{3}{25} + 0 \times \frac{1}{100} = \frac{240}{100} = 2.4 \text{ 分}$$

17. 解析: (1) $\because AB=BC=1, \angle ABC=60^\circ, \therefore \triangle ABC$ 为正三角形 $AC=1$,

又 $AB \parallel CD, \therefore \angle ACD = \angle BAC = 60^\circ$

$$\text{又 } CD=2, \therefore AD^2 = AC^2 + CD^2 - 2AC \cdot CD \cdot \cos 60^\circ = 1 + 4 - 2 \times 1 \times 2 \times \frac{1}{2} = 3$$

$$\therefore AD^2 + AC^2 = CD^2, \therefore AD \perp AC,$$

又 $AD \perp AM, AM, AC$ 为平面 AMC 内两条相交直线,

又 $AD \subset$ 平面 $MAC, AC \subset$ 平面 $ABCD, \therefore$ 平面 $MAC \perp$ 平面 $ABCD$

(2) 取 AC 中点 O, CD 中点 $F, \because AM=CM, \therefore MO \perp AC$,

由(1)知平面 $MAC \perp$ 平面 $ABCD$, 面 $MAC \cap$ 面 $ABCD = AC$,

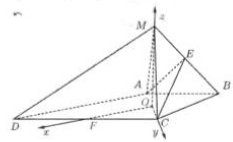
$\therefore MO \perp$ 平面 $ABCD$, 又 $OF \perp AC$,

故以 O 为原点建立如图所示空间直角坐标系, $C\left(0, \frac{1}{2}, 0\right), A\left(0, -\frac{1}{2}, 0\right), M\left(0, 0, \frac{3}{2}\right)$,

$$F\left(\frac{\sqrt{3}}{2}, 0, 0\right), D\left(\sqrt{3}, -\frac{1}{2}, 0\right), B\left(-\frac{\sqrt{3}}{2}, 0, 0\right)$$

$$\text{设 } E(x, y, z), \text{ 由 } \vec{BE} = \lambda \vec{EM} \text{ 得 } \begin{cases} x + \frac{\sqrt{3}}{2} = \lambda x \\ y - \frac{1}{2} = \lambda y \\ z = \lambda \left(-x - y - \frac{3}{2} - z\right) \end{cases}$$

$$\begin{cases} x = -\frac{\sqrt{3}}{2(1+\lambda)} \\ y = 0 \\ z = \frac{3\lambda}{2(1+\lambda)} \end{cases} \therefore E\left(-\frac{\sqrt{3}}{2(1+\lambda)}, 0, \frac{3\lambda}{2(1+\lambda)}\right)$$



设平面 ADM 的法向量为 $\vec{m} = (x_1, y_1, z_1)$

$$\text{则 } \begin{cases} \vec{m} \cdot \vec{AD} = 0 \\ \vec{m} \cdot \vec{AM} = 0 \end{cases} \Rightarrow \begin{cases} (x_1, y_1, z_1) \cdot (\sqrt{3}, 0, 0) = 0 \\ (x_1, y_1, z_1) \cdot \left(0, \frac{1}{2}, \frac{3}{2}\right) = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ \frac{1}{2}y_1 + \frac{3}{2}z_1 = 0 \end{cases} \text{ 取 } \vec{m} = (0, 3, -1)$$

设平面 ACE 的法向量为 $\vec{n} = (x_2, y_2, z_2)$

$$\begin{cases} \vec{n} \cdot \vec{CA} = 0 \\ \vec{n} \cdot \vec{CE} = 0 \end{cases} \Rightarrow \begin{cases} (x_2, y_2, z_2) \cdot (0, -1, 0) = 0 \\ (x_2, y_2, z_2) \cdot \left(-\frac{\sqrt{3}}{2(1+\lambda)}, -\frac{1}{2}, \frac{3\lambda}{2(1+\lambda)} \right) = 0 \end{cases} \Rightarrow \begin{cases} y_2 = 0 \\ -\sqrt{3}x_2 + 3\lambda z_2 = 0 \end{cases}$$

取 $\vec{n} = (\sqrt{3}\lambda, 0, 1)$, 则 $\vec{m} \cdot \vec{n} = (0, 3, -1) \cdot (\sqrt{3}\lambda, 0, 1) = -1$, $|\vec{m}| = \sqrt{10}$, $|\vec{n}| = \sqrt{3\lambda^2 + 1}$
由 $\frac{|\vec{m} \cdot \vec{n}|}{|\vec{m}||\vec{n}|} = \frac{\sqrt{70}}{70}$ 得: $\frac{1}{\sqrt{10} \cdot \sqrt{3\lambda^2 + 1}} = \frac{\sqrt{70}}{70}$, $3\lambda^2 + 1 = 7$
 $\therefore \lambda^2 = 2$, 又 $\lambda > 0$, $\therefore \lambda = \sqrt{2}$

18. 解析: (1) 由题意知, $e_1 = e_2$, $\therefore \frac{b^2}{a^2} = \frac{4}{12}$, $\therefore a^2 = 3b^2$, 又 $\because P(\sqrt{3}, 1)$ 在椭圆上, $\therefore \frac{3}{a^2} + \frac{1}{b^2} = 1$,

$$\therefore b^2 = 2, a^2 = 6, \therefore \text{椭圆 } E_1 \text{ 的标准方程为 } \frac{x^2}{6} + \frac{y^2}{2} = 1$$

(2) 要证 $S_{\triangle APQ} = S_{\triangle BQP}$ 即证 $|AP| = |BQ|$, 设 $A(x_1, y_1), B(x_2, y_2), P(x_p, y_p), Q(x_q, y_q)$

① 当直线 l_1 斜率不存在时, 由椭圆对称性可知 $AP = BQ$ 成立

② 当直线 l_1 斜率存在时, 设 k_1 , 则 AB 方程: $y - 1 = k_1(x - \sqrt{3})$

$$\text{联立 } \begin{cases} y - 1 = k_1(x - \sqrt{3}) \\ \frac{x^2}{6} + \frac{y^2}{2} = 1 \end{cases} \text{ 得 } (3k_1^2 + 1)x^2 + (6k_1 - 6\sqrt{3}k_1^2)x + 3(1 - \sqrt{3}k_1)^2 - 6 = 0$$

$$\therefore x_p + x_q = \frac{6\sqrt{3}k_1^2 - 6k_1}{3k_1^2 + 1}, x_p x_q = \frac{3(1 - \sqrt{3}k_1)^2 - 6}{3k_1^2 + 1}$$

$$\text{联立 } \begin{cases} y - 1 = k_1(x - \sqrt{3}) \\ \frac{x^2}{12} + \frac{y^2}{4} = 1 \end{cases} \text{ 得 } (3k_1^2 + 1)x^2 + (6k_1 - 6\sqrt{3}k_1^2)x + 3(1 - \sqrt{3}k_1)^2 - 12 = 0$$

$$\therefore x_1 + x_2 = \frac{6\sqrt{3}k_1^2 - 6k_1}{3k_1^2 + 1}, x_1 x_2 = \frac{3(1 - \sqrt{3}k_1)^2 - 12}{3k_1^2 + 1}$$

$$\therefore x_p + x_q = x_1 + x_2, \therefore x_p - x_1 = x_2 - x_q$$

$$|AP| = \sqrt{1 + k_1^2} \cdot |x_p - x_1|, |BQ| = \sqrt{1 + k_1^2} \cdot |x_2 - x_q| \quad \therefore |AP| = |BQ|$$

综上所述: $S_{\triangle APQ} = S_{\triangle BQP}$

(3) 由第二问可知 $|CP| = |DH|$, $\therefore \frac{|BQ|}{|DH|} = \frac{|DP|}{|BP|} \Rightarrow |AP| \cdot |BP| = |CP| \cdot |DP|$

设直线 CD 的斜率为 k_2 , 直线 CD 方程为 $y - 1 = k_2(x - \sqrt{3})$, 设 $C(x_c, y_c), D(x_d, y_d)$

$$\text{联立 } \begin{cases} y - 1 = k_2(x - \sqrt{3}) \\ \frac{x^2}{12} + \frac{y^2}{4} = 1 \end{cases} \text{ 得 } (3k_2^2 + 1)x^2 - (6\sqrt{3}k_2^2 - 6k_2)x + 3(1 - \sqrt{3}k_2)^2 - 12 = 0$$

$$\therefore x_c + x_d = \frac{6\sqrt{3}k_2^2 - 6k_2}{3k_2^2 + 1}, x_c x_d = \frac{3(1 - \sqrt{3}k_2)^2 - 12}{3k_2^2 + 1}$$

$$|CP| \cdot |DP| = \sqrt{1 + k_2^2} |x_c - x_p| \sqrt{1 + k_2^2} |x_d - x_p|$$

$$|AP| \cdot |BP| = \sqrt{1 + k_1^2} |x_1 - x_p| \sqrt{1 + k_1^2} |x_2 - x_p|$$

$$\therefore (1 + k_2^2)(x_c - x_p)(x_d - x_p) = (1 + k_1^2)(x_1 - x_p)(x_2 - x_p)$$

$$\text{即 } (1 + k_2^2)[x_c x_d - x_p(x_c + x_d) + x_p^2] = (1 + k_1^2)[x_1 x_2 - x_p(x_1 + x_2) + x_p^2]$$

$$\text{化简得 } \frac{1 + k_2^2}{3k_2^2 + 1} = \frac{1 + k_1^2}{3k_1^2 + 1}, \therefore k_1^2 = k_2^2, \text{ 由题意 } k_1 \neq k_2, \therefore k_1 + k_2 = 0, \therefore \alpha + \beta = \pi$$

19. 解析: (1) $f(x)$ 定义域为 $x \in (-1, +\infty)$, $f'(x) = \frac{1}{x+1} - m$

① 当 $-m \geq 0$ 即 $m \leq 0$ 时, $f'(x) > 0$, $f(x)$ 在 $(-1, +\infty)$ 上为增函数;

② 当 $-m < 0$ 即 $m > 0$ 时, $f'(x) = \frac{-m(x+1-\frac{1}{m})}{x+1} = 0 \Leftrightarrow x = \frac{1}{m} - 1 > -1$,

$$f'(x) > 0 \Leftrightarrow -1 < x < \frac{1}{m} - 1,$$

$\therefore f(x)$ 在 $(-1, \frac{1}{m} - 1)$ 上为增函数, 在 $(\frac{1}{m} - 1, +\infty)$ 上为减函数

(2) $m = 1$ 时, $h(x) = \cos x + \ln(x+1) - x$, $h'(x) = -\sin x + \frac{1}{x+1} - 1$

① $x \in (-1, 0]$ 时, $h'(x)$ 在 $[-1, 0]$ 上单调递减, $\therefore h'(x) \geq h'(0) = 0$

$\therefore h(x)$ 在 $(-1, 0]$ 上单调递增, 又 $h(0) = 1 > 0$, $h(\frac{1}{e^2} - 1) = \cos(\frac{1}{e^2} - 1) - 2 - \frac{1}{e^2} + 1 < 0$

$\therefore \exists x_1 \in (\frac{1}{e^2} - 1, 0)$, 使得 $h(x_1) = 0$, 即 $h(x)$ 在 $(-1, 0]$ 上有且仅有 1 个零点 x_1

② $x \in [\pi, +\infty)$ 时, 由(1)知 $f(x) = \ln(x+1) - x$ 在 $[\pi, +\infty)$ 上单调递减,

$$\text{即 } f(x) \leq f(\pi) = \ln(\pi+1) - \pi$$

$$\therefore h(x) = \cos x + f(x) \leq 1 + \ln(\pi+1) - \pi < 1 + \ln e^2 - \pi = 3 - \pi < 0$$

$\therefore h(x)$ 在 $[\pi, +\infty)$ 上没有零点

③ $x \in (0, \pi)$ 时, $\begin{cases} -\sin x < 0 \\ \frac{1}{x+1} - 1 < 0 \end{cases}, \therefore h'(x) = -\sin x + \frac{1}{x+1} - 1 < 0$

即 $h(x)$ 在 $(0, \pi)$ 上单调递减, 又 $h(0) = 1$, $h(\pi) = \ln(\pi+1) - \pi - 1 < 0$

$\therefore h(x)$ 在 $(0, \pi)$ 上有且仅有一个零点 x_2

综上所述, $h(x)$ 在 $(-1, +\infty)$ 上有且仅有两个不同的零点 x_1 和 x_2

(3) 令 $\varphi(x) = f(x) + g(x) = \ln(x+1) + \cos mx - mx - 1$

由于 $\varphi(x) \leq 0$ 恒成立, 且 $\varphi(0) = 0$, 同时 $\varphi(x)$ 在 $(-1, +\infty)$ 上连续,



$\therefore x=0$ 是 $\varphi(x)$ 的一个极大值点

$\therefore \varphi'(x) = \frac{1}{x+1} - m \sin mx - m$, $\therefore \varphi'(0) = 1 - m = 0$ 即 $m = 1$.

下面证明 $m=1$ 时 $\varphi(x) \leq 0$ 在 $x \in (-1, +\infty)$ 上恒成立

由(1)知 $m=1$ 时, $f(x)$ 在 $(-1, 0)$ 上单调递增, 在 $(0, +\infty)$ 上单调递减

$\therefore f(x) \leq f(0) = 0$, 又 $g(x) = \cos x - 1 \leq 0$, $\therefore \varphi(x) = f(x) + g(x) \leq 0$ 恒成立

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