

## 2022-2023 学年第二学期高二年级期中质量监测

### 数学试题参考答案及评分标准

#### 一、选择题

题号	1	2	3	4	5	6	7	8
答案	D	C	A	C	D	D	B	B

#### 二、选择题

题号	9	10	11	12
答案	AD	BCD	BC	ACD

#### 三、填空题

13、58      14、0.03      15、0.62      16、 $\frac{75}{512}$

#### 四、解答题

17、(8分)

解：(1) 因为前三项的二项式系数之和等于 79，

$$\text{所以 } C_n^0 + C_n^1 + C_n^2 = 1 + n + \frac{n(n-1)}{2} = 79, \dots\dots\dots 2 \text{ 分}$$

$$\text{解得 } n = 12 \text{ 或 } n = -13. \dots\dots\dots 3 \text{ 分}$$

$$\text{因为 } n > 0, \text{ 所以 } n = 12. \dots\dots\dots 4 \text{ 分}$$

$$(2) \left(ax + \frac{1}{\sqrt[3]{x}}\right)^{12} \text{ 的通项为 } T_{r+1} = C_{12}^r (ax)^{12-r} \left(\frac{1}{\sqrt[3]{x}}\right)^r = C_{12}^r a^{12-r} x^{12-\frac{4r}{3}}, \dots\dots\dots 6 \text{ 分}$$

$$\text{所以当 } 12 - \frac{4r}{3} = 0 \text{ 时, } r = 9, \dots\dots\dots 7 \text{ 分}$$

$$\text{此时, 常数项为 } C_{12}^9 a^3 = \frac{55}{2},$$

$$\text{解得 } a = \frac{1}{2}. \dots\dots\dots 8 \text{ 分}$$

18、(10分)

解：(1) 零假设为

$H_0$ : 数学成绩与语文成绩无关, 据表中数据计算得

$$\chi^2 = \frac{200(50 \times 80 - 30 \times 40)^2}{90 \times 110 \times 120 \times 80} \approx 16.498 > 6.635, \dots\dots\dots 4 \text{ 分}$$

根据  $\alpha = 0.01$  的独立性检验, 我们推断  $H_0$  不成立, 认为数学成绩与语文成绩有关.  $\dots\dots\dots 5$  分

(2)

$$L(B|A) = \frac{P(B|A)}{P(\bar{B}|A)} = \frac{\frac{P(AB)}{P(A)}}{\frac{P(A\bar{B})}{P(A)}} = \frac{P(AB)}{P(A\bar{B})} = \frac{80}{30} = \frac{8}{3}.$$

$\therefore$  估计  $L(B|A)$  的值为  $\frac{8}{3}$ .  $\dots\dots\dots 10$  分

19、(10分)

解: (1)  $\mu = \frac{1}{8}(20+12+13+15+16+14+12+18) = 15, \dots\dots\dots 2$  分

$$\sigma = \sqrt{\frac{1}{8}(25+9+4+0+1+1+9+9)} = \sqrt{\frac{58}{8}} = \frac{\sqrt{29}}{2}. \dots\dots\dots 5$$
 分

(2)  $\mu - 3\sigma = 15 - \frac{3\sqrt{29}}{2}, \mu + 3\sigma = 15 + \frac{3\sqrt{29}}{2},$

则  $(\mu - 3\sigma, \mu + 3\sigma) = (15 - \frac{3\sqrt{29}}{2}, 15 + \frac{3\sqrt{29}}{2}), \dots\dots\dots 8$

分

因为 12, 13, 14, 15, 16, 18, 20 均属于  $(\mu - 3\sigma, \mu + 3\sigma)$ , 所以各点都不是孤立点.  $\dots\dots\dots 10$  分

20A、(10分)

解: (1) 记该同学前三道题答对  $k$  道为事件  $A_k$ , 第四道答对为事件  $B$ ,

$$P(A_k) = C_3^k \left(\frac{4}{5}\right)^k \left(\frac{1}{5}\right)^{3-k}, k \in \{0, 1, 2, 3\}$$

$$P(X = 15) = P(A_3 \bar{B}) + P(A_2 B) = \left(\frac{4}{5}\right)^3 \left(\frac{1}{2}\right) + C_3^2 \left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right) \left(\frac{1}{2}\right) = \frac{56}{125}, \dots\dots\dots 2$$
 分

$$P(X = 20) = P(A_3 B) = \left(\frac{4}{5}\right)^3 \left(\frac{1}{2}\right) = \frac{32}{125}, \dots\dots\dots 3$$
 分

$$\therefore P(X \geq 15) = \frac{56}{125} + \frac{32}{125} = \frac{88}{125}. \dots\dots\dots 4$$
 分

(2)  $X$  的取值可能为 0, 5, 10, 15, 20,

$$P(X=0) = P(A_0\bar{B}) = \left(\frac{1}{5}\right)^3 \left(\frac{1}{2}\right) = \frac{1}{250},$$

$$P(X=5) = P(A_1\bar{B}) + P(A_0B) = C_3^1 \left(\frac{4}{5}\right) \left(\frac{1}{5}\right)^2 \left(\frac{1}{2}\right) + \left(\frac{1}{5}\right)^3 \left(\frac{1}{2}\right) = \frac{13}{250},$$

$$P(X=10) = P(A_2\bar{B}) + P(A_1B) = C_3^2 \left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right) \left(\frac{1}{2}\right) + C_3^1 \left(\frac{4}{5}\right) \left(\frac{1}{5}\right)^2 \left(\frac{1}{2}\right) = \frac{6}{25},$$

$$P(X=15) = P(A_3\bar{B}) + P(A_2B) = \left(\frac{4}{5}\right)^3 \left(\frac{1}{2}\right) + C_3^2 \left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right) \left(\frac{1}{2}\right) = \frac{56}{125},$$

$$P(X=20) = P(A_3B) = \left(\frac{4}{5}\right)^3 \left(\frac{1}{2}\right) = \frac{32}{125}.$$

则  $X$  的分布列为:

X	0	5	10	15	20
P	$\frac{1}{250}$	$\frac{13}{250}$	$\frac{6}{25}$	$\frac{56}{125}$	$\frac{32}{125}$

自主选拔在线  
微信号: z1225w

.....8分

$$E(X) = 0 \times \frac{1}{250} + 5 \times \frac{13}{250} + 10 \times \frac{6}{25} + 15 \times \frac{56}{125} + 20 \times \frac{32}{125} = 14.5.$$

该同学填空题得分的均值是 14.5 分. ....10分

20B、(10分)

(1) 记该同学前两道题答对  $k$  道为事件  $A_k$ , 第三道答对为事件  $B$ , 第四道答对为事件  $C$ ,

$$\text{则 } P(X \geq 15) = P(A_2)P(B+C) + P(A_1BC) = P(A_2)[1 - P(\bar{B}\bar{C})] + P(A_1BC)$$

$$= \left(\frac{5}{6}\right)^2 \left(1 - \frac{1}{2} \times \frac{2}{3}\right) + C_2^1 \left(\frac{5}{6}\right) \left(\frac{1}{6}\right) \left(\frac{1}{2}\right) \left(\frac{1}{3}\right) = \frac{55}{108} \dots\dots\dots 4 \text{分}$$

(2)  $X$  的取值可能为 0, 5, 10, 15, 20,

$$P(X=0) = P(A_0\bar{B}\bar{C}) = \left(\frac{1}{6}\right)^2 \left(\frac{1}{2}\right) \left(\frac{2}{3}\right) = \frac{2}{216} = \frac{1}{108},$$

$$P(X=5) = P(A_1\bar{B}\bar{C}) + P(A_0B\bar{C}) + P(A_0\bar{B}C) = C_2^1 \left(\frac{5}{6}\right) \times \frac{1}{6} \times \frac{1}{2} \times \frac{2}{3} + \left(\frac{1}{6}\right)^2 \times \frac{1}{2} \times \frac{2}{3} + \left(\frac{1}{6}\right)^2 \times \frac{1}{2} \times \frac{1}{3} = \frac{23}{216},$$

$$P(X=10) = P(A_2\bar{B}\bar{C}) + P(A_1B\bar{C}) + P(A_1\bar{B}C) + P(A_0BC)$$

$$= \left(\frac{5}{6}\right)^2 \times \frac{1}{2} \times \frac{2}{3} + C_2^1 \left(\frac{5}{6}\right) \times \frac{1}{6} \times \frac{1}{2} \times \frac{2}{3} + C_2^1 \left(\frac{5}{6}\right) \times \frac{1}{6} \times \frac{1}{2} \times \frac{1}{3} + \left(\frac{1}{6}\right)^2 \times \frac{1}{2} \times \frac{1}{3} = \frac{81}{216} = \frac{3}{8},$$

$$P(X=15) = P(A_2\bar{B}C) + P(A_2B\bar{C}) + P(A_1BC) = \left(\frac{5}{6}\right)^2 \times \frac{1}{2} \times \frac{1}{3} + \left(\frac{5}{6}\right)^2 \times \frac{1}{2} \times \frac{2}{3} + C_2^1 \left(\frac{5}{6}\right) \times \frac{1}{6} \times \frac{1}{2} \times \frac{1}{3} = \frac{85}{216},$$

$$P(X=20) = P(A_2BC) = \left(\frac{5}{6}\right)^2 \times \frac{1}{2} \times \frac{1}{3} = \frac{25}{216},$$

则  $X$  的分布列为:

X	0	5	10	15	20
P	$\frac{1}{108}$	$\frac{23}{216}$	$\frac{3}{8}$	$\frac{85}{216}$	$\frac{25}{216}$

.....8分

$$E(X) = \frac{1}{216}(0 \times 2 + 5 \times 23 + 10 \times 81 + 15 \times 85 + 20 \times 25) = 12.5.$$

该同学填空题得分的均值是 12.5 分. ....10分

21A、(10分)

解: (1)由题意得  $\bar{x} = \frac{7+6+5+4+3}{5} = 5, \bar{y} = \frac{2.5+2.3+1.8+1.9+1.5}{5} = 2,$

$$\sum_{i=1}^5 (x_i - \bar{x})(y_i - \bar{y}) = 2 \times 0.5 + 1 \times 0.3 + 0 \times 0.2 + 1 \times 0.1 + 2 \times 0.5 = 2.4,$$

$$\text{又 } \sum_{i=1}^5 (x_i - \bar{x})^2 = 2^2 + 1^2 + 0^2 + 1^2 + 2^2 = 10,$$

$$\therefore \hat{b} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = 0.24, \quad \hat{a} = \bar{y} - \hat{b}\bar{x} = 2 - 0.24 \times 5 = 0.8, \dots\dots\dots 4 \text{分}$$

所以  $y$  关于  $x$  的线性回归方程为  $y = 0.24x + 0.8$ . ....5分

(2) 设前者和后者的斜率分别为  $k_1, k_2$

$$\text{由 } \sum_{i=1}^5 (y_i - \bar{y})^2 = 0.5^2 + 0.3^2 + 0.2^2 + 0.1^2 + 0.5^2 = 0.64,$$

$$\text{得 } k_2 = \frac{\sum_{i=1}^5 (y_i - \bar{y})^2}{\sum_{i=1}^5 (x_i - \bar{x})(y_i - \bar{y})} = \frac{0.64}{2.4} = \frac{4}{15} \approx 0.27, \dots\dots\dots 8 \text{分}$$

由 (1) 知  $k_1 = \hat{b} = 0.24,$

$\therefore k_1 < k_2$ . ....10分

21B、(10分)

解析: (1)  $\bar{x} = \frac{1}{15} \sum_{i=1}^{15} x_i = 5$ ,  $\bar{y} = \frac{1}{15} \sum_{i=1}^{15} y_i = 2$ ,

$\hat{b} = \frac{9}{30} = 0.3$ ,  $\hat{a} = 2 - 0.3 \times 5 = 0.5$ , .....4分

故回归方程为  $\hat{y} = 0.3x + 0.5$ . .....5分

(2) 设前者和后者的斜率分别为  $k_1, k_2$

$$k_1 = \hat{b} = \frac{\sum_{i=1}^{15} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{15} (x_i - \bar{x})^2}, \quad k_2 = \frac{1}{\hat{b}} = \frac{\sum_{i=1}^{15} (y_i - \bar{y})^2}{\sum_{i=1}^{15} (x_i - \bar{x})(y_i - \bar{y})}$$

$$r^2 = \frac{\left( \sum_{i=1}^{15} (x_i - \bar{x})(y_i - \bar{y}) \right)^2}{\sum_{i=1}^{15} (x_i - \bar{x})^2 \sum_{i=1}^{15} (y_i - \bar{y})^2} = \frac{k_1}{k_2}, \quad \text{即 } k_1 = r^2 k_2 \quad \dots\dots\dots 7 \text{分}$$

①显然有  $0 < r < 1$ , 故  $k_1 < k_2$ , 即前者斜率小于后者. ....8分

②注意到, 两直线都过  $(\bar{x}, \bar{y})$ , 且  $k_1 < k_2$ , 故公共点仅有  $(5, 2)$ . ....10分