

汕头市一模参考答案与评分标准

1	2	3	4	5	6	7	8	9	10	11	12
C	D	C	B	A	D	A	B	BC	ACD	ABD	ABC

三、填空题：本题共4小题，每小题5分，共20分。

13. 解： $(x + \frac{2}{x} - y)^{10}$ 的展开式的通项公式为 $T_{r+1} = C_{10}^r (x + \frac{2}{x})^{10-r} \cdot (-y)^r$

$r=7, T_8 = C_{10}^7 (x + \frac{2}{x})^3 \cdot (-y)^7 = C_{10}^7 C_3^4 x^{3-4} \cdot 2^4 x^{-4} \cdot (-y)^7 = C_{10}^7 C_3^4 \cdot 2^4 x^{3-24} \cdot (-y)^7$

令 $3-24=1$, 则 $k=1$, 展开式中 xy^7 的系数为 $C_{10}^7 C_3^4 2^4 \cdot (-1)^7$. 故答案为 -720 .

14. 令 $x > 0$, 则 $-x < 0$, 又 $\because f(-x) = f(x)$ 且 $x > 0$ 时 $f(x) = e^x - 1 \therefore f(x) = f(-x) = e^{-x} - 1$

当 $x < 0$ 时, $f'(x) = -e^{-x}, f'(-1) = -e, f(-1) = e - 1$ 所求切线 $y - (e - 1) = -e(x + 1)$ 即 $ex + y + 1 = 0$

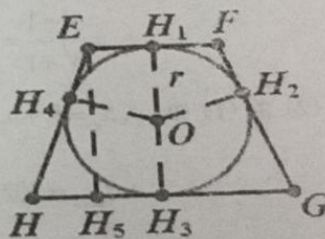
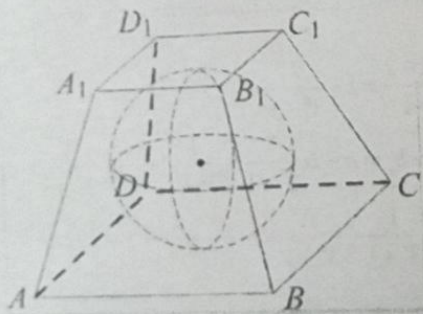
15. 解：如图，作该正棱台的轴截面，其中 E, F, M, N 分别是 AB, CD, C_1D_1, A_1B_1 的中点， H, K 是 MN, EF 的中点， G 是内切球的球心， H, K 是内切球和上、下底面的切点， Q 是内切球和侧面 CDD_1C_1 的切点，

内切球的半径为 r ，由在正四棱台 $ABCD - A_1B_1C_1D_1$ 中， $AB = 4, A_1B_1 = 2$ ，则 $HM = 1, KF = 2$ ，

$HG = KG = QG = r$ ，易得 $MQ = HM = 1, FQ = FK = 2$ ，则 $MF = MQ + QF = 3, MG^2 = 1^2 + r^2$ ，

$FG^2 = 2^2 + r^2$ ，且 $\angle MGF = 90^\circ$ ，所以 $MG^2 + FG^2 = MF^2$ ，即 $1 + r^2 + 4 + r^2 = 9$ ，解得 $r = \sqrt{2}$ 。

从而可知该球的表面积 $S = 4\pi r^2 = 8\pi$ 。



16. 解：双曲线 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 (a > 0, b > 0)$ 的渐近线方程为 $bx \pm ay = 0$ 。设点 $P(x_0, y_0)$ ，可得

$$y - y_0 = \pm \frac{b}{a}(x - x_0), \text{ 分别联立 } \begin{cases} y - y_0 = \frac{b}{a}(x - x_0) \\ y = \frac{b}{a}x \end{cases} \quad \begin{cases} y - y_0 = -\frac{b}{a}(x - x_0) \\ y = \frac{b}{a}x \end{cases}$$

$$\text{可得 } M\left(\frac{bx_0 + ay_0}{2b}, \frac{bx_0 + ay_0}{2a}\right), N\left(\frac{bx_0 - ay_0}{2b}, \frac{bx_0 - ay_0}{2a}\right), \overline{OM} \cdot \overline{ON} = \frac{b^2x_0^2 - a^2y_0^2}{4b^2} - \frac{b^2x_0^2 - a^2y_0^2}{4a^2}$$

$$\frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1, \therefore b^2x_0^2 - a^2y_0^2 = a^2b^2$$

$$\therefore \overline{OM} \cdot \overline{ON} = \frac{a^2 - b^2}{4}, \text{ 由题意 } \frac{a^2 - b^2}{4} \geq \frac{b^2}{4}, \text{ 所以 } a^2 \geq 2b^2, \text{ 即 } 1 < \frac{b^2}{a^2} < \frac{1}{2}, \text{ 所以 } 1 < 1 +$$

$$\frac{b^2}{a^2} \leq \frac{3}{2}, \text{ 即 } 1 < e^2 \leq \frac{3}{2}, \text{ 故 } e \in \left(1, \frac{\sqrt{6}}{2}\right]$$

17. (1) 由正弦定理得 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r, \dots\dots\dots (1 \text{ 分})$

在 $\triangle ABD$ 中, $\frac{BD}{\sin \alpha} = \frac{AB}{\sin \angle ADB}$, 在 $\triangle ACD$ 中, $\frac{CD}{\sin \beta} = \frac{AC}{\sin \angle ADC} \dots\dots\dots (2 \text{ 分})$

$\because \angle ADB + \angle ADC = \pi, \therefore \sin \angle ADB = \sin \angle ADC, \dots\dots\dots (3 \text{ 分})$

$\therefore \frac{AB \cdot \sin \alpha}{BD} = \frac{AC \cdot \sin \beta}{CD}, \therefore \frac{BD}{CD} = \frac{AB \cdot \sin \alpha}{AC \cdot \sin \beta} \dots\dots\dots (5 \text{ 分})$

(2) 法一: 由(1)知, $\frac{BD}{DC} = \frac{AB \cdot \sin \alpha}{AC \cdot \sin \beta} = \frac{2\sqrt{7}}{2 \sin 90^\circ} = \sqrt{7} \sin \alpha = \frac{1}{2} \therefore \sin \alpha = \frac{1}{2} \times \frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{14} \dots\dots\dots (6 \text{ 分})$

$$\cos(\alpha + \beta) = \cos(\alpha + 90^\circ) = -\sin \alpha = -\frac{\sqrt{7}}{14} \dots\dots\dots (7 \text{ 分})$$

$$\overline{AD} = \frac{2}{3}\overline{AB} + \frac{1}{3}\overline{AC}$$

$$(\overline{AD})^2 = \left(\frac{2}{3}\overline{AB} + \frac{1}{3}\overline{AC}\right)^2 = \frac{4}{9}\overline{AB}^2 + \frac{4}{9}\overline{AC}^2 + \frac{4}{9}\overline{AB} \cdot \overline{AC} = \frac{4}{9} \times (2\sqrt{7})^2 + \frac{1}{9} \times 4 + \frac{4}{9} \times 2\sqrt{7} \times 2 \cos(\alpha + \beta) = 12$$

$$\therefore AD = 2\sqrt{3} \dots\dots\dots (9 \text{ 分})$$

$$\therefore S_{\triangle ACD} = \frac{1}{2} AD \cdot AC = 2\sqrt{3} \dots\dots\dots (10 \text{ 分})$$

法二: 由(1)知, $\frac{S_{\triangle ABD}}{S_{\triangle ACD}} = \frac{BD}{DC} = \frac{AB \cdot \sin \alpha}{AC \cdot \sin \beta} = \frac{2\sqrt{7}}{2 \sin 90^\circ} = \sqrt{7} \sin \alpha = \frac{1}{2}$,

$$\therefore \sin \alpha = \frac{1}{2} \times \frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{14} \dots\dots\dots (6 \text{ 分})$$

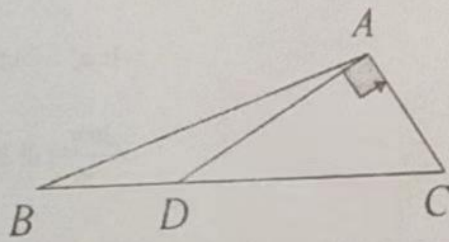
$\because \angle BAC$ 为钝角, $\beta = 90^\circ, \therefore \alpha \in \left(0, \frac{\pi}{2}\right), \cos \alpha = \sqrt{1 - \sin^2 \alpha} = \frac{3\sqrt{21}}{14} \dots\dots\dots (7 \text{ 分})$

$$\therefore S_{\triangle ACD} = \frac{2}{3} S_{\triangle ABC} = \frac{2}{3} \times \frac{1}{2} \times AB \times AC \times \sin \angle BAC = \frac{2}{3} \times \frac{1}{2} \times 2\sqrt{7} \times 2 \times \frac{3\sqrt{21}}{14} = 2\sqrt{3} \dots\dots\dots (10 \text{ 分})$$

$\dots\dots\dots (6 \text{ 分})$

法三: (2) 依题意, 设 $BD = x$, 则 $CD = 2x, BC = 3x$

在 $\triangle ACD$ 中, $\beta = 90^\circ, AD = \sqrt{CD^2 - AC^2} = \sqrt{4 - 4x^2} \quad \cos C = \frac{AC}{CD} = \frac{2}{2x} = \frac{1}{x} \dots\dots\dots (7 \text{ 分})$



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