

哈师大附中 2022 年高三第三次模拟考试

文科数学答案

一、选择题

1-6: B A B C D A 7-12: D A C D B D

二、填空题

13. $\frac{2\pi}{3}$ 14. $2\sqrt{2}$ 15. 25π 16. $\frac{\sqrt{3}}{2}$

1-16 详解:

1. 【解析】 $A = (-\infty, 1), B = (0, 2), A \cap B = (0, 1)$, 故选 B.

2. 【解析】 $z = \frac{2i}{1+i^3} = \frac{2i(1+i)}{(1-i)(1+i)} = -1+i$, 则 $\bar{z} = -1-i$, 故选 A

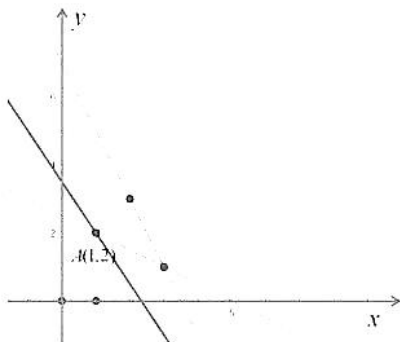
3. 【解析】 $\cos \alpha + \sin \alpha = \frac{3\sqrt{2}}{5}$ (1). 平方得 $2 \sin \alpha \cos \alpha = -\frac{7}{25}$

$\sin \alpha - \cos \alpha = \sqrt{1 - 2 \sin \alpha \cos \alpha} = \frac{4\sqrt{2}}{5}$ (2). (1)×(2)得, $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = -\frac{24}{25}$, 故选 B.

4. 【解析】 根据系统抽样定义知每个学生入选的概率都为 $P = \frac{20}{702} = \frac{10}{351}$, 故选 C

5. 【解析】 “平面 α 与平面 β 不垂直”, 不能推出“直线 a 与直线 b 不垂直”, 反之也推不出, 故选 D

6. 【解析】 如图, 当 $x=1, y=2$ 时, $z_{\min} = 3 \times 1 + 2 \times 2 = 7$, 故选 A



7. 【解析】 本题考查的是古典概型. 设一块中型三角形为 a , 两块小型三角形为 b_1, b_2 , 两块大型三角形为 c_1, c_2 , 则样本空间为 $\{(a, b_1), (a, b_2), (a, c_1), (a, c_2), (b_1, b_2), (b_1, c_1), (b_1, c_2), (b_2, c_1), (b_2, c_2), (c_1, c_2)\}$, 共 10 个基本事件, 两块板恰好是全等三角形有 $\{(b_1, b_2), (c_1, c_2)\}$, 共 2 个基本事件, 所以所求概率为 $P = \frac{2}{10} = \frac{1}{5}$, 故选 D

8. 【解析】设所有棱长为1，延长 A_1B_1 至点 G ，使 $|A_1B_1| = |B_1G|$ ，取线段 B_1G 的中点 F ，

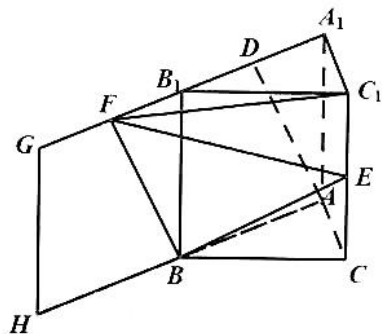
$\therefore DF \parallel AB, DF = AB$ ， \therefore 四边形 $ABFD$ 为平行四边形， $\therefore AD \parallel BF$ ，

$\therefore \angle EBF$ 为异面直线 AD, BE 所成角或其补角。

$$\therefore BE = BF = \frac{\sqrt{5}}{2}, C_1F^2 = 1 + \frac{1}{4} - 2 \times 1 \times \frac{1}{2} \times \left(-\frac{1}{2}\right) = \frac{7}{4}$$

$$\therefore EF = \sqrt{C_1E^2 + C_1F^2} = \sqrt{2}$$

$$\therefore \cos \angle EBF = \frac{\frac{5}{4} + \frac{5}{4} - 2}{2 \times \frac{\sqrt{5}}{2} \times \frac{\sqrt{5}}{2}} = \frac{1}{5}, \text{ 异面直线 } AD, BE \text{ 所成角的余弦值为 } \frac{1}{5} \text{ 故选 A}$$



9. 【解析】 $\frac{T}{2} \geq \frac{\pi}{6} - \left(-\frac{\pi}{3}\right) = \frac{\pi}{2}, \therefore T \geq \pi, \therefore 0 < \omega \leq 2$

由已知 $f\left(\frac{\pi}{6}\right) = 1, f\left(-\frac{\pi}{3}\right) = 0$

$$\text{则 } \begin{cases} \omega \cdot \frac{\pi}{6} + \varphi = 2k_1\pi + \frac{\pi}{2} \\ \omega \cdot \left(-\frac{\pi}{3}\right) + \varphi = 2k_2\pi \end{cases}, k_1, k_2 \in \mathbf{Z}, \therefore \omega = 4(k_1 - k_2) + 1, k_1, k_2 \in \mathbf{Z}, \therefore 0 < \omega \leq 2, \therefore \omega = 1,$$

$\therefore \varphi = 2k_1\pi + \frac{\pi}{3}, k_1 \in \mathbf{Z}, \therefore |\varphi| < \frac{\pi}{2}, \therefore \varphi = \frac{\pi}{3}$, 故选 C

10. 【解析】

$$\begin{aligned} f(x) + f(-x) &= \frac{2^{x+1}}{2^x + 1} + \frac{2^{x-1}}{2^x - 1} + \frac{2^{-x-1}}{2^{-x} + 1} + \frac{2^{-x-1}}{2^{-x} - 1} \\ &= \frac{2^{x+1}}{2^x + 1} + \frac{2^{x-1}}{2^x - 1} + \frac{2}{2^x + 1} - \frac{1}{2^x - 1} = \frac{2(2^x + 1)}{2^x + 1} + \frac{2^{x-1}(2^x - 1)}{2^x - 1} = \frac{5}{2} \end{aligned}$$

原式 $= f(\ln 2) + f(-\ln 2) = \frac{5}{2}$, 故选 D.

11. 【解析】 $\therefore \begin{cases} |PF_1| + |PF_2| = 6 \\ |PF_1| = 2|PF_2| \end{cases} \therefore |PF_1| = 4, |PF_2| = 2 \therefore |F_1F_2| = 4 \therefore \triangle PF_1F_2$ 为等腰三角形.

$$\therefore S_{\triangle PF_1F_2} = \frac{1}{2} \times 2 \times \sqrt{4^2 - 1^2} = \sqrt{15},$$

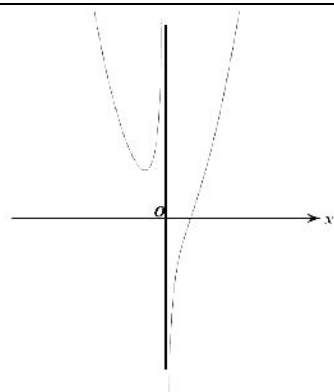
设 $\triangle PF_1F_2$ 内切圆半径为 r , 则 $S_{\triangle PF_1F_2} = \frac{1}{2}(6+4)r = \sqrt{15} \therefore r = \frac{\sqrt{15}}{5}$, 故选 B

12. 【解析】 $\because x \neq 0 \therefore a = \frac{x^3 - 1}{x} = x^2 - \frac{1}{x}$

设 $g(x) = x^2 - \frac{1}{x} \therefore g'(x) = 2x + \frac{1}{x^2} = \frac{2x^3 + 1}{x^2}$, $g(x)$ 在 $(-\infty, -\sqrt[3]{\frac{1}{2}})$ 上递减,

在 $(-\sqrt[3]{\frac{1}{2}}, 0)$, $(0, +\infty)$ 上递增.

如图所示, 若 $a = \frac{x^3 - 1}{x} = x^2 - \frac{1}{x}$ 有唯一实数解, $a < g(-\sqrt[3]{\frac{1}{2}}) = \frac{3}{2}\sqrt[3]{2}$, 故选 D



13. 【解析】 $(2a + b) \cdot a = 2a^2 + a \cdot b = 2 + 4 \cos \langle a, b \rangle = 0$, 则 $\cos \langle a, b \rangle = -\frac{1}{2}$, $\because 0 \leq \langle a, b \rangle \leq \pi$, $\therefore \langle a, b \rangle = \frac{2\pi}{3}$

14. 【解析】设 $A(x_1, y_1), B(x_2, y_2)$

$$\begin{cases} y = x - \frac{p}{2} \\ y^2 = 2px \end{cases} \Rightarrow x^2 - 3px + \frac{p^2}{4} = 0 \quad \therefore \Delta > 0, x_1 + x_2 = 3p, x_1 x_2 = \frac{p^2}{4}$$

$|AB| = x_1 + x_2 + p = 4p = 8 \therefore p = 2$.

所以直线方程为 $x - y - 1 = 0$, 原点到直线的距离是 $\frac{\sqrt{2}}{2}$

$\therefore S_{\triangle AOB} = \frac{1}{2} \times 8 \times \frac{\sqrt{2}}{2} = 2\sqrt{2}$

15. 【解析】设 $\triangle ABC$ 的外接圆圆心为 O_1 , 半径为 r , 球的半径为 R , $OO_1 = d$.

则 $OO_1 \perp$ 平面 ABC , 当球心 O 在线段 PO_1 上时, $\left(\frac{r}{R}\right)_{\max} = \frac{R+d}{d} = 4 \therefore R = 3d$

$2r = \frac{10}{\sin 45^\circ} = \frac{10\sqrt{2}}{3} \therefore r = \frac{5\sqrt{2}}{3}$

$\because d^2 = R^2 - r^2 \therefore R^2 = 9d^2 = 9(R^2 - r^2), \therefore R^2 = \frac{9}{8}r^2 = \frac{9}{8} \times \frac{50}{9} = \frac{25}{4}, \therefore 4\pi R^2 = 25\pi$

16. 【解析】 $\frac{2b+c}{a \cos C} = \frac{4}{2} = 2$, 由正弦定理, $\frac{2 \sin B + \sin C}{\sin A \cos C} = 2, 2 \sin B + \sin C = 2 \sin A \cos C$,

$2 \sin(A+C) + \sin C = 2 \sin A \cos C + 2 \cos A \sin C + \sin C = 2 \sin A \cos C$, 解得 $\cos A = -\frac{1}{2}$, 即 $A = \frac{2\pi}{3}$.

$4 = 2b+c \geq 2\sqrt{2bc}$, $bc \leq 2$ (当 $2b=c$ 时取等号)

$S_{\triangle ABC} = \frac{1}{2}bc \sin A = \frac{\sqrt{3}}{4}bc \leq \frac{\sqrt{3}}{2}$, 故 $\triangle ABC$ 面积最大值为 $\frac{\sqrt{3}}{2}$.

三、解答题

17. 解: (1) $\because a_n + S_n = 1, a_{n-1} + S_{n-1} = 1 (n \geq 2)$

两式相减得 $a_n + S_n - a_{n-1} - S_{n-1} = 2a_n - a_{n-1} = 0 (n \geq 2)$,

$\therefore a_1 + S_1 = 2a_1 = 1, \therefore a_1 = \frac{1}{2} \neq 0$

$\therefore \frac{a_n}{a_{n-1}} = \frac{1}{2} (n \geq 2)$, 所以数列 $\{a_n\}$ 是以 $\frac{1}{2}$ 为首项和公比的等比数列, -----4 分

因此, $a_n = \left(\frac{1}{2}\right)^n$; -----6 分

(2) $b_n = \left(\frac{1}{2}\right)^n + \log_2 \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^n - n$ -----7 分

$T_n = \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \dots + \left(\frac{1}{2}\right)^n - (1 + 2 + \dots + n)$

$= \frac{\frac{1}{2} \left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} - \frac{n(n+1)}{2} = \frac{2 - n^2 - n}{2} - \frac{1}{2^n}$ -----12 分

18. 解: (1) 根据散点图, 模型二更适宜作为人工植树成活数与年份代码变化关系的回归分析模型; -----2 分

(2) 设 $z = \ln y, \hat{z} = \hat{\lambda} + \hat{\mu}x$

$\bar{x} = 5.5, \bar{z} = 2.5, \sum_{i=1}^{10} x_i^2 = 385, \sum_{i=1}^{10} x_i z_i = 150.7$

$\hat{\mu} = \frac{150.7 - 10 \times 5.5 \times 2.5}{385 - 10 \times 5.5^2} = \frac{13.2}{82.5} = 0.16$ -----5 分

$\hat{\lambda} = \bar{z} - \hat{\mu}\bar{x} = 2.5 - 0.16 \times 5.5 = 1.62$ -----6 分

$\therefore \hat{z} = \ln \hat{y} = 0.16x + 1.62$,

因此, 根据 (1) 选定的模型, y 关于 x 的回归方程是 $\hat{y} = e^{0.16x+1.62}$; -----8 分

(3) 令 $\hat{y} = e^{0.16x+1.62} > 50, \therefore 0.16x + 1.62 > \ln 50 = \ln 2 + \ln 25 = \ln 2 + 2 \ln 5 \approx 3.911$ -----10 分

$x > 14.31875$, 预测从 2025 年开始人工植树成活棵数能够超过 5 万棵. -----12 分

19. (1) 证明: 取 AP 中点 N,

\because PB 的中点为 M, $\therefore MN \parallel \frac{1}{2}AB$, $MN = \frac{1}{2}AB$

$\because CD \parallel \frac{1}{2}AB$, $CD = \frac{1}{2}AB$

$\therefore MN \parallel CD$, 且 $MN=CD$, \therefore 四边形 CDNM 是平行四边形,

$\therefore CM \parallel DN$ -----3 分

$\because CM \not\subset$ 平面 PAD

$\therefore CM \parallel$ 平面 PAD -----5 分

(2) (方法一) 解: 等腰梯形 ABCD 中, $AB=2, CD=1, \angle ABC=60^\circ$, $\therefore BC=1$,

$\triangle ABC$ 中, 由余弦定理得 $AC = \sqrt{3}$, $\therefore AC^2 + BC^2 = AB^2$, 即 $AC \perp CB$

$\because PC \perp$ 底面 ABCD, $\therefore PC \perp AC$

$\because CB \cap PC = C$, $\therefore AC \perp$ 平面 PBC, -----8 分

$\therefore A-PCM$ 的高为 $AC = \sqrt{3}$

由 M 是 PB 的中点, $S_{\triangle PCM} = \frac{1}{2} S_{\triangle PCB} = \frac{1}{2} \times \frac{1}{2} BC \cdot PC = \frac{1}{2} \times \frac{1}{2} \times 1 \times 3 = \frac{3}{4}$ -----10 分

$\therefore V_{P-ACM} = V_{A-PCM} = \frac{1}{3} \times S_{\triangle PCM} \times AC = \frac{1}{3} \times \frac{3}{4} \times \sqrt{3} = \frac{\sqrt{3}}{4}$ -----12 分

(方法二) 解: 等腰梯形 ABCD 中, $AB=2, CD=1, \angle ABC=60^\circ$, $\therefore BC=1$.

$\triangle ABC$ 中, 由余弦定理得 $AC = \sqrt{3}$, $\therefore AC^2 + BC^2 = AB^2$, 即 $AC \perp CB$

$\because PC \perp$ 底面 ABCD, $\therefore PC \perp AC$

$\because CB \cap PC = C$, $\therefore AC \perp$ 平面 PBC, -----8 分

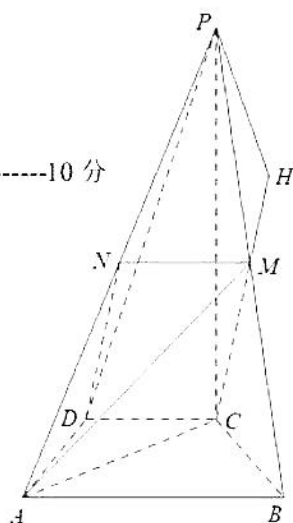
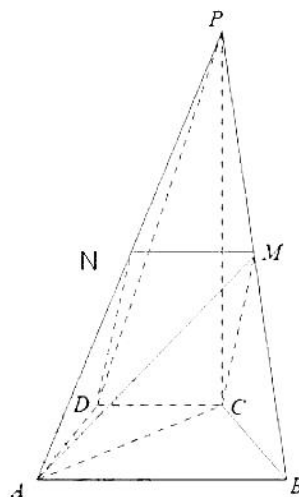
作 $PH \perp CM$ 于 H, $PH \subset$ 平面 PBC, 则 $AC \perp PH$.

$\because AC \cap CM = C$, $\therefore PH \perp$ 平面 ACM,

由 M 是 PB 的中点, $PH = \frac{PC \times \frac{1}{2}BC}{CM} = \frac{3 \times \frac{1}{2}}{\frac{\sqrt{10}}{2}} = \frac{3\sqrt{10}}{10}$ 为 P-ACM 的高 -----10 分

$S_{\triangle ACM} = \frac{1}{2} CM \cdot AC = \frac{1}{2} \times \frac{\sqrt{10}}{2} \times \sqrt{3} = \frac{\sqrt{30}}{4}$, -----11 分

$\therefore V_{P-ACM} = \frac{1}{3} S_{\triangle ACM} \cdot PH = \frac{1}{3} \times \frac{\sqrt{30}}{4} \times \frac{3\sqrt{10}}{10} = \frac{\sqrt{3}}{4}$ -----12 分



20. 解: (1) 设直线 ET 的方程为: $y = k(x+4), k > 0$

$$\begin{cases} y = k(x+4) \\ x^2 + 4y^2 = 8 \end{cases}, (4k^2 + 1)x^2 + 32k^2x + 64k^2 - 8 = 0$$

$$\Delta = (32k^2)^2 - 4(4k^2 + 1)(64k^2 - 8) = 32(1 - 4k^2) = 0 \quad \text{-----2分}$$

$$\because k > 0 \therefore k = \frac{1}{2}, \therefore x^2 + 4x + 4 = 0, \therefore x = -2$$

$$\therefore T(-2, 1) \quad \text{-----4分}$$

(2) $\because E(-4, 0), T(-2, 1), \therefore$ 中点 $G(-3, \frac{1}{2})$

(i) 直线 l 的斜率为 $-\frac{1}{2}$ 时, 方程为 $y - \frac{1}{2} = -\frac{1}{2}(x+3)$, 即 $y = -\frac{1}{2}x - 1$

$$\begin{cases} y = -\frac{1}{2}x - 1 \\ x^2 + 4y^2 = 8 \end{cases}, \text{得 } x^2 + 2x - 2 = 0, x_1 = -1 - \sqrt{3}, x_2 = -1 + \sqrt{3}$$

不妨设 $A(-1 - \sqrt{3}, \frac{\sqrt{3}-1}{2}), B(-1 + \sqrt{3}, \frac{-1-\sqrt{3}}{2})$ -----6分

$$k_{EA} = \frac{\frac{1}{2}(\sqrt{3}-1)}{3-\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}, \text{ 直线 } EA \text{ 的方程: } y = \frac{\sqrt{3}}{6}(x+4)$$

$$\begin{cases} y = \frac{\sqrt{3}}{6}(x+4) \\ x^2 + 4y^2 = 8 \end{cases}, x^2 + 2x - 2 = 0 \therefore x_M = -1 + \sqrt{3}, \therefore y_M = \frac{\sqrt{3}}{6}(x_M + 4) = \frac{1 + \sqrt{3}}{2}$$

$$\therefore M(-1 + \sqrt{3}, \frac{1 + \sqrt{3}}{2}) \quad \text{-----8分}$$

同理, $k_{EB} = -\frac{\sqrt{3}}{6}$, 直线 EB 的方程: $y = -\frac{\sqrt{3}}{6}(x+4)$,

$$\begin{cases} y = -\frac{\sqrt{3}}{6}(x+4) \\ x^2 + 4y^2 = 8 \end{cases}, x^2 + 2x - 2 = 0 \therefore x_N = -1 - \sqrt{3} \therefore y_N = -\frac{\sqrt{3}}{6}(x_N + 4) = \frac{1 - \sqrt{3}}{2}$$

$$\therefore N(-1 - \sqrt{3}, \frac{1 - \sqrt{3}}{2}) \quad \text{-----10分}$$

$$\therefore k_{MN} = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2} \quad \text{-----11分}$$

(ii) $MN // ET$ -----12分

21. (1) 解: $\because f(x) = x \ln x + a(\ln x - 2x + 2), \therefore f'(x) = 1 + \ln x + a(\frac{1}{x} - 2),$

记 $g(x) = f'(x) = 1 + \ln x + a(\frac{1}{x} - 2),$ 则 $g'(x) = \frac{x-a}{x^2}$

① $a \leq 1$ 时, 区间 $(1, +\infty)$ 上, $x-a > 0, g'(x) > 0, f'(x)$ 单调递增,

$f'(x) > f'(1) = 1-a \geq 0, \therefore f(x)$ 单调递增, $\therefore f(x) > f(1) = 0, \therefore a \leq 1$ 满足条件. -----2分

② $a > 1$ 时,

区间 $(1, a)$ 上, $x-a < 0, g'(x) < 0, f'(x)$ 单调递减, $f'(x) < f'(1) = 1-a < 0,$

$\therefore f(x)$ 单调递减, $f(x) < f(1) = 0,$ 与 $f(x) > 0$ 矛盾, $\therefore a > 1$ 不满足条件. -----4分

由 ①②, 实数 a 的取值范围是 $(-\infty, 1]$. -----5分

(2) (方法一)

由 (1) 知 $a = 0$ 时, $f'(x) = 1 + \ln x, f'(x)$ 在 $(0, +\infty)$ 上单调递增, 又 $f'(\frac{1}{e}) = 0,$

$\therefore (0, \frac{1}{e})$ 上 $f'(x) < 0, f(x)$ 单调递减, $(\frac{1}{e}, +\infty)$ 上 $f'(x) > 0, f(x)$ 单调递增,

$f(x) \geq f(\frac{1}{e}) = -\frac{1}{e},$ 即 $x \ln x \geq -\frac{1}{e},$ -----8分

$\therefore x^2 \ln x^2 \geq -\frac{1}{e}$ -----10分

$\therefore x^2 \ln x^2 + x \ln x \geq -\frac{2}{e},$ 即 $2x^2 \ln x + x \ln x \geq -\frac{2}{e},$

$\therefore (2x^2 + x) \ln x \geq -\frac{2}{e} > -1, \therefore x > 0, \therefore (2x+1) \ln x > \frac{-1}{x},$

$\therefore (2x+1) \ln x + \frac{1}{x} > 0.$ -----12分

(方法二) 证明: $(2x+1) \ln x + \frac{1}{x} > 0 \Leftrightarrow (2x^2+x) \ln x > -1.$

记 $g(x) = (2x^2+x) \ln x, g'(x) = (4x+1) \ln x + (2x+1).$

记 $h(x) = g'(x) = (4x+1) \ln x + (2x+1), h'(x) = 4 \ln x + \frac{1}{x} + 6,$

记 $m(x) = h'(x) = 4 \ln x + \frac{1}{x} + 6, m'(x) = \frac{4x-1}{x^2},$

$(0, \frac{1}{4})$ 上, $m'(x) < 0, m(x)$ 单调递减, $(\frac{1}{4}, +\infty)$ 上, $m'(x) > 0, m(x)$ 单调递增,

$m(x) = h'(x) = 4 \ln x + \frac{1}{x} + 6 \geq h'(\frac{1}{4}) = 10 - 8 \ln 2 > 10 - 8 \ln e = 2 > 0, \therefore h(x) = g'(x)$ 在 $(0, +\infty)$ 上单调递增, -----7分

$\therefore g'(\frac{1}{2}) = 2 - 3 \ln 2 = \ln e^2 - \ln 8 < 0, g'(\frac{1}{\sqrt{e}}) = \frac{1}{2} > 0,$

$\therefore \exists x_0 \in (\frac{1}{2}, \frac{1}{\sqrt{e}})$, 使得 $g'(x_0) = 0$, 即 $(4x_0 + 1)\ln x_0 + (2x_0 + 1) = 0$,

$(0, x_0)$ 上, $g'(x) < 0$, $g(x)$ 单调递减, $(x_0, +\infty)$ 上, $g'(x) > 0$, $g(x)$ 单调递增,

$$g(x) = (2x^2 + x)\ln x \geq g(x_0) = (2x_0^2 + x_0)\ln x_0 = (2x_0^2 + x_0) \cdot \left(-\frac{2x_0 + 1}{4x_0 + 1}\right), \quad \text{-----9分}$$

$$\text{记 } L(x_0) = (2x_0^2 + x_0) \cdot \left(-\frac{2x_0 + 1}{4x_0 + 1}\right), L'(x_0) = \frac{-32x_0^3 - 28x_0^2 - 8x_0 - 1}{(4x_0 + 1)^2} < 0,$$

$$\therefore L(x_0) \text{ 在 } \left(\frac{1}{2}, \frac{1}{\sqrt{e}}\right) \text{ 上单调递减, } \therefore L(x_0) > L\left(\frac{1}{\sqrt{e}}\right) = \left(2\left(\frac{1}{\sqrt{e}}\right)^2 + \left(\frac{1}{\sqrt{e}}\right)\right) \cdot \left(-\frac{2\left(\frac{1}{\sqrt{e}}\right) + 1}{4\left(\frac{1}{\sqrt{e}}\right) + 1}\right) = -\frac{\frac{4}{e} + \frac{4}{\sqrt{e}} + 1}{4 + \sqrt{e}},$$

-----10分

$$\therefore g(x)_{\min} = L(x_0) > L\left(\frac{1}{\sqrt{e}}\right) = -\frac{\frac{4}{e} + \frac{4}{\sqrt{e}} + 1}{4 + \sqrt{e}}$$

$$-\frac{\frac{4}{e} + \frac{4}{\sqrt{e}} + 1}{4 + \sqrt{e}} > -1 \Leftrightarrow 4 - 3e + (4 - e)\sqrt{e} < 0 \Leftrightarrow 4 - 3e + (4 - e) \times 2 < 0 \Leftrightarrow 12 - 5e < 0 \text{ (成立)}$$

$$\therefore g(x) > -1, \text{ 即 } (2x^2 + x)\ln x > -1, \therefore (2x + 1)\ln x + \frac{1}{x} > 0. \quad \text{-----12分}$$

22. 解: (1) $C_1: x^2 - y^2 = 4 (x \geq 2)$ -----2分

所以曲线 C_1 的极坐标方程为 $\rho^2 \cos^2 \theta - \rho^2 \sin^2 \theta = 4 \left(-\frac{\pi}{4} < \theta < \frac{\pi}{4}\right)$ -----4分

(2) 曲线 C_2 的极坐标方程为 $\theta = \frac{\pi}{6} (\rho > 0)$ -----5分

将 $\theta = \frac{\pi}{6}$ 代入 C_1, C_3 的极坐标方程分别得:

$$|OA| = \frac{2}{\sqrt{\cos^2 \frac{\pi}{6} - \sin^2 \frac{\pi}{6}}} = 2\sqrt{2} \quad \text{-----7分}$$

$$|OB| = 4\sqrt{2} \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) - 2\cos \frac{\pi}{6} = 2 + \sqrt{3} \quad \text{-----9分}$$

$$\therefore |AB| = |OB| - |OA| = 2 + \sqrt{3} - 2\sqrt{2}. \quad \text{-----10分}$$

23. 解: (1) $\because f(x) = \begin{cases} -3x, x \leq -\frac{1}{2}, \\ 2+x, -\frac{1}{2} < x < 1, \\ 3x, x > 1, \end{cases}$

$\therefore |2x+1| + |x-1| > 4 \Leftrightarrow \begin{cases} x \leq -\frac{1}{2} & \text{或} & \begin{cases} -\frac{1}{2} < x < 1 \\ 2+x > 4 \end{cases} & \text{或} & \begin{cases} x \geq 1 \\ 3x > 4 \end{cases} \end{cases}$ -----3分

所以原不等式的解集为 $(-\infty, -\frac{4}{3}) \cup (\frac{4}{3}, +\infty)$. -----5分

(2) 若 $-2 \leq x \leq -\frac{1}{2}$, 则 $f(x) = -3x \leq f(-2) = 6$;

若 $-\frac{1}{2} < x < 1$, 则 $f(x) = x+2 < f(1) = 3$;

若 $1 < x \leq 2$, 则 $f(x) = 3x \leq f(2) = 6$

$\therefore x \in [-2, 2]$ 时, $f(x)_{\max} = f(-2) = f(2) = 6$ -----7分

$\therefore |a-4| < 6 \therefore 2 < a < 10$.

实数 a 的取值范围是 $(2, 10)$ -----10分



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