高三一模数学答案

一．单选题 1.C　2.D　3.B　4.A　5.D　6.A　7.D　8.B

二．多选题 9.ABC　10. AC　11. ABD　12.　BC

三．填空题 13. 40　14. $\frac{64\sqrt{3}}{27}$　15.　[0，e）　16.　2　11

四．解答题

17.(1)$\because PD//$面α，面$\alpha\cap$面$PCD=EF$，$PD\subset$面$PCD,\therefore PD//EF$　………3

(2)$\because PA\perp$面$ABCD,\therefore\angle PCA=45^\circ,\therefore PA=AC=2.$　………4

由(1)得F为PC中点.$\because$在菱形$ABCD$中$\angle BAD=120^\circ$ E为中点$\therefore AE\perp CD.$

以A为坐标原点，以AB,AE,AP所在直线为x,y,z轴建立空间直角坐标系

$P(0,0,2),C(1,\sqrt{3},0),E(0,\sqrt{3},0),D(-1,\sqrt{3},0),F\left(\frac{1}{2},\frac{\sqrt{3}}{2},1\right)$

设面AEF法向量$\vec{m}=(x_1,y_1,z_1)$，$\overrightarrow{AE}=(0,\sqrt{3},0),\overrightarrow{AF}=\left(\frac{1}{2},\frac{\sqrt{3}}{2},1\right),\therefore\begin{cases}\sqrt{3}y_1=0\\ \frac{1}{2}x_1+\frac{\sqrt{3}}{2}y_1+z_1=0\end{cases}\therefore\vec{m}=(2,0,-1)$ ……6

设面EFD法向量为$\vec{n}=(x_2,y_2,z_2),\overrightarrow{ED}=(-1,0,0),\overrightarrow{EF}=\left(\frac{1}{2},-\frac{\sqrt{3}}{2},1\right),\therefore\begin{cases}\sqrt{3}x_2=0\\ \frac{1}{2}x_2-\frac{\sqrt{3}}{2}y_2+z_2=0\end{cases}\therefore\vec{n}=(0,2,\sqrt{3})$ …8

设二面角$A-EF-D$的大小为$\theta,\therefore\cos\theta=\frac{\vec{m}\cdot\vec{n}}{|\vec{m}|\cdot|\vec{n}|}=\frac{\sqrt{105}}{35},$　……9

$\therefore\sin\theta=\frac{4\sqrt{70}}{35}$　……10

18.(1)当$n=2$时，$a_2=2$. ……1

当$n\ge 3$时，$S_{n-1}+S_{n-2}=a_{n-1}{}^2$，$\therefore S_n-S_{n-2}=a_n{}^2-a_{n-1}{}^2$

$\therefore a_n+a_{n-1}=(a_n+a_{n-1})(a_n-a_{n-1})$，$\because$ 正项数列中，$a_n+a_{n-1}>0$，$\therefore a_n-a_{n-1}=1$ ……3

$\because a_2-a_1=1$，$\therefore$ 数列$\{a_n\}$是首项为$a_1=1$，公差$d=1$的等差数列， ……4

$\therefore a_n=n$ ……5

(2)由(1)$b_n=\dfrac{1}{2^n}+\dfrac{2}{2^{n-1}}+\cdots+\dfrac{n}{2}$

$2^n b_n=1\cdot 2^0+2\cdot 2^1+\cdots+n\cdot 2^{n-1}$记为$T_n$

$2T_n=1\cdot 2^1+2\cdot 2^2+\cdots+n\cdot 2^n \quad \therefore -T_n=2^0+2^1+\cdots+2^{n-1}-n\cdot 2^n$ ……7

$-T_n=\dfrac{1-2^n}{1-2}-n\cdot 2^n \quad \therefore T_n=(n-1)2^n+1$. ……9

$\therefore b_n=\dfrac{(n-1)2^n+1}{2^n}=n-1+\dfrac{1}{2^n}$. ……10

$\therefore b_n-\lambda n+1=\dfrac{1}{2^n}+(1-\lambda)n$，设$c_n=\dfrac{1}{2^n}+(1-\lambda)n$，则$c_2{}^2=c_1c_3$，解得$\lambda=1$

$\therefore \dfrac{c_n}{c_{n-1}}=\dfrac{\frac{1}{2^n}}{\frac{1}{2^{n-1}}}=\dfrac{1}{2}$，$\therefore$ 数列$\{b_n-\lambda n+1\}$是等比数列， ……11

则$\lambda=1$ ……12

19.(1)由余弦定理$\cos A=\dfrac{b^2+c^2-a^2}{2bc}=\dfrac{c^2-bc}{2bc}$，$\therefore c-b=2b\cos A$

由正弦定理$\sin C-\sin B=2\sin B\cos A$，又$C=\pi-(A+B)$，$\therefore \sin C=\sin(A+B)$

代入化简得$\sin(A-B)=\sin B$. $\therefore A-B=B$或$A-B+B=\pi$(舍)，$\therefore A=2B$ ……3

$\because \Delta ABC$为锐角三角形，$\therefore \begin{cases} 0<B<\dfrac{\pi}{2} \\ 0<A=2B<\dfrac{\pi}{2} \\ 0<C=\pi-3B<\dfrac{\pi}{2} \end{cases}$ 得$B\in\left(\dfrac{\pi}{6},\dfrac{\pi}{4}\right)$ ……5

(2)设AB上的高为h，$\because S=\dfrac{1}{2}ch=\dfrac{1}{2}ac\sin B$，$\therefore h=a\sin B$ ……6

由正弦定理$\dfrac{a}{\sin 2B}=\dfrac{b}{\sin B}=\dfrac{c}{\sin(\pi-3B)}=\dfrac{c}{\sin 3B}$，$\therefore a=\dfrac{4\sin 2B}{\sin 3B}$ ……8

$\therefore h=a\sin B=\dfrac{4\sin 2B\cdot\sin B}{\sin 2B\cos B+\cos 2B\sin B}=\dfrac{4}{\dfrac{1}{\tan B}+\dfrac{1}{\tan 2B}}=\dfrac{8\tan B}{3-\tan^2 B}=\dfrac{8}{\dfrac{3}{\tan B}-\tan B}$ ……10

$\because \dfrac{3}{\tan B}-\tan B$在$\left(\dfrac{\pi}{6},\dfrac{\pi}{4}\right)$单调递减，$\therefore \dfrac{3}{\tan B}-\tan B\in\left(2,\dfrac{8\sqrt{3}}{3}\right)$，$\therefore h\in(\sqrt{3},4)$ ……12

20.(1)设事件$A=$该同学在一轮比赛中获得巧手奖 ……1

$P(A)=\frac{C_3^1\cdot C_2^2+C_3^2\cdot C_2^1\cdot C_2^1+C_3^2C_2^2}{C_4^2C_4^2}=\frac{1}{2}$, ……3

则该同学在一轮比赛中获得巧手奖的概率为$\frac{1}{2}$ ……4

(2)设强化训练后规定作品入选的概率P_1，创意作品入选概率为P_2，则$P_1+P_2=\frac{3}{4}+\frac{1}{2}+\frac{1}{4}=\frac{3}{2}$ ……5

所以强化训练后该同学一轮比赛中可获得巧手奖的概率

$P=C_2^1P_1(1-P_1)\cdot P_2^2+C_2^1P_2(1-P_2)\cdot P_1^2+P_1^2P_2^2=3P_1P_2-3(P_1P_2)^2$

$\because P_1+P_2=\frac{3}{2}$, $P_1>\frac{3}{4}, P_2>\frac{1}{2}, \therefore P_1=\frac{3}{2}-P_2>\frac{3}{4}, P_2=\frac{3}{2}-P_1>\frac{1}{2}$ 得$P_1\in\left(\frac{3}{4},1\right), P_2\in\left(\frac{1}{2},\frac{9}{4}\right)$

$\therefore P_1P_2=P_1\left(\frac{3}{2}-P_1\right)=-P_1^2+\frac{3}{2}P_1\in\left(\frac{1}{2},\frac{9}{16}\right)$

$\therefore P=3P_1P_2-3(P_1P_2)^2<\frac{3}{4}$ ……10

$\because$四轮比赛中获得巧手奖的次数$X\sim B(4，P)$, ……11

$\therefore E(X)=4P<3, \therefore$ 该同学不能进入决赛. ……12

21.(1)方法一：设过$E(x_0,y_0)$与l_2平行的直线为$y-y_0=-(x-x_0)$.由$\begin{cases}y=x\\ y-y_0=-(x-x_0)\end{cases}$得$x_A=\frac{x_0+y_0}{2}$,……1

$\therefore |OA|=\sqrt{2}\left|\frac{x_0+y_0}{2}\right|$. E到直线l_1的距离为$\frac{|x_0-y_0|}{\sqrt{2}}$ ……3

$\therefore \sqrt{2}\left|\frac{x_0+y_0}{2}\right|\cdot\frac{|x_0-y_0|}{\sqrt{2}}=1$, 化简得$x_0^2-y_0^2=\pm 2$, 所以$E$的轨迹方程为$\frac{x^2}{2}-\frac{y^2}{2}=1$或$\frac{y^2}{2}-\frac{x^2}{2}=1$ ……5

方法二：由已知$l_1\perp l_2$,可得四边形$OAEB$是矩形， ……1

$\therefore E$到l_1的距离$d_1=\frac{|x_0-y_0|}{\sqrt{2}}$， E到l_2的距离$d_2=\frac{|x_0+y_0|}{\sqrt{2}}$, ……3

$\therefore S=\frac{|x_0-y_0|}{\sqrt{2}}\cdot\frac{|x_0+y_0|}{\sqrt{2}}=1$， 化简得$E$的轨迹方程为$\frac{x^2}{2}-\frac{y^2}{2}=1$或$\frac{y^2}{2}-\frac{x^2}{2}=1$ ……5(少解扣一分)

(2)$E_0:\frac{x^2}{2}-\frac{y^2}{2}=1$, 设$l:x=my-2$, 联立$\begin{cases}x=my-2\\ \frac{x^2}{2}-\frac{y^2}{2}=1\end{cases}$ 得$(m^2-1)y^2-4my+2=0, \Delta>0$ 恒成立 且$m^2\neq 1$……6

设$P(x_1,y_1),Q(x_2,y_2)$ 由韦达定理$y_1+y_2=\frac{4m}{m^2-1}, y_1y_2=\frac{2}{m^2-1}$ ……7

$\therefore x_1+x_2=m(y_1+y_2)-4=\frac{4}{m^2-1}$. $\because \Delta PQH$重心为O, $\therefore H(-(x_1+x_2),-(y_1+y_2))=\left(-\frac{4}{m^2-1},-\frac{4m}{m^2-1}\right)$…9

又H在H_0上，代入得$m^4+6m^2-7=0$ 解得$m^2=1$(舍)或$m^2=-7$(舍), ……11

综上，不存在这样的H点 ……12

22.(1)$x\in(0,+\infty)$. $f'(x)=m(x-1)-2+\frac{1}{x}=\frac{mx^2-(m+2)x+1}{x}$. ……1

令$h(x)=mx^2-(m+2)x+1\because m\ge 1$,且$\Delta=m^2+4>0$, ……2

$\therefore h(x)=mx^2-(m+2)x+1=0$有两个不等实根$a,b$.

又$m\ge 1$,且$h(0)=1>0$,$\therefore h(x)=mx^2-(m+2)x+1<0$在$(0,+\infty)$上的解集为$(a,b)$,

即$f(x)$存在单调递减区间(a,b).由韦达定理$a+b=\frac{m+2}{m}$, $ab=\frac{1}{m}$

$\therefore b-a=\sqrt{(b+a)^2-4ab}=\sqrt{1+\frac{4}{m^2}}$ ……4

$\because m\ge 1,\therefore b-a\in\left(1,\sqrt{5}\right]$ ……5

(2)令$g(x)=\frac{1}{2}m(x-1)^2+x+\ln x-xe^{x-1}\le 0$在$[1,+\infty)$上恒成立

$g'(x)=m(x-1)+1+\frac{1}{x}-(x+1)e^{x-1}$，令$G(x)=m(x-1)+1+\frac{1}{x}-(x+1)e^{x-1}$,

$G'(x)=m-\frac{1}{x^2}-(x+2)e^{x-1}$，令$\varphi(x)=m-\frac{1}{x^2}-(x+2)e^{x-1}$,则$\varphi'(x)=\frac{2}{x^3}-(x+3)e^{x-1}$,

令$\psi(x)=\frac{2}{x^3}-(x+3)e^{x-1}$，则$\psi'(x)=-\frac{6}{x^4}-(x+4)e^{x-1}<0$

$\therefore\psi(x)$在$[1,+\infty)$上单调递减，$\psi(x)\le\psi(1)=2-4<0$,

$\therefore\varphi(x)$在$[1,+\infty)$上单调递减，$\varphi(x)\le\varphi(1)=m-1-3=m-4$. ……7

当$m-4\le 0$，即$m\le 4$时，$G(x)=g'(x)$在$[1,+\infty)$上单调递减，$g'(x)\le g'(1)=0$,

此时$g(x)$在$[1,+\infty)$上单调递减,$g(x)\le g(1)=0$,成立,$\therefore 1\le m\le 4$ ……9

当$m-4>0$，即$m>4$时，$\varphi(x)=0$在$[1,+\infty)$上有根设为x_0,

$\therefore g'(x)$在$[1，x_0)$上单调递增，在$(x_0,+\infty)$上单调递减，且$g'(1)=0$

设$g'(x)=0$在$[1,+\infty)$上有根设为x_1,

$\therefore g(x)$在$[1，x_1)$上单调递增，在$(x_1,+\infty)$上单调递减,$\therefore g(x)_{\max}=g(x_1)>g(1)=0$

此时不成立，舍去, ……11

综上所述,$1\le m\le 4$. ……12