

高三阶段性考试 数学参考答案(理科)

1. A $A \cap B = \{x \mid -1 < x < 0\}$.
2. B 若“ $x \in A$ ”是“ $x \in B$ ”的充分不必要条件,则 $A \subsetneq B$,故选 B.
3. C 由题意可知 $3 \mid \frac{b}{2} - 8$,所以 $p = 10$.
4. D 因为 $a > \log_2 \sqrt{5} = \frac{1}{2}$, $b = 2^{-1/2} < 2^{-1} = \frac{1}{2}$,所以 $a > b$.
因为 $a = \log_2 3 < \log_2 5 = 1$, $c = 0.3^{-0.2} > 0.3^0 = 1$,所以 $a < c$,故 $c > a > b$.
5. D 因为 $f(x) = \sin \omega x + \sqrt{3} \cos \omega x = 2\sin(\omega x + \frac{\pi}{3})$,且最小正周期为 π ,所以 $\omega = 2$,即 $f(x) = 2\sin(2x + \frac{\pi}{3})$.
由题意可知 $g(x) = 2\sin[2(x + \frac{\pi}{6}) + \frac{\pi}{3}] = 2\sin(2x + \frac{2\pi}{3})$,因为 $g(\frac{\pi}{6}) = 0$,所以 $g(x)$ 的图像关于点 $(\frac{\pi}{6}, 0)$ 对称.
6. B 设 BC 边的中点为 E,则 $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AE}$,则 $\overrightarrow{AD} = \sqrt{3} \frac{|\overrightarrow{BC}|}{2} \cdot \overrightarrow{AE}$.
故 $\frac{|\overrightarrow{AD}|}{|\overrightarrow{BC}|} = \frac{\sqrt{3} |\overrightarrow{AE}|}{|\overrightarrow{BC}|} = \frac{\sqrt{3} \times \frac{\sqrt{3}}{2} |\overrightarrow{BC}|}{|\overrightarrow{BC}|} = \frac{3}{2}$.
7. C 因为 $a + 4b = 6$,所以 $(a-1) + 4(b-1) = 1$.
因为 $\frac{1}{a-1} + \frac{1}{b-1} = (\frac{1}{a-1} + \frac{1}{b-1})[(a-1) + 4(b-1)] = 5 + \frac{4(b-1)}{a-1} + \frac{a-1}{b-1} \geq 5 + 2\sqrt{\frac{4(b-1)}{a-1} \cdot \frac{a-1}{b-1}} = 9$,当且仅当 $\frac{4(b-1)}{a-1} = \frac{a-1}{b-1}$,即 $a = \frac{4}{3}$, $b = \frac{7}{6}$ 时,等号成立.
8. D 由题意知 $c = m - 1 = m = 1$,所以 $c = 1$,又因为 $\frac{1}{\sqrt{m+1}} = \frac{1}{2}$,所以 $m = 3$,解得 $a = 2$, $c = 1$.因为 $b = \sqrt{a^2 - c^2} = \sqrt{3}$,所以 C 的短轴长为 $2\sqrt{3}$.
9. C 如图,设 $PO = h$ 米,则 $\frac{h}{OA} = \tan 60^\circ$, $\frac{h}{OB} = \tan 50^\circ$.
所以 $AB = OB - OA = h(\frac{1}{\tan 50^\circ} - \frac{1}{\tan 60^\circ})$.
则 $h = \frac{AB \tan 50^\circ \tan 60^\circ}{\tan 60^\circ - \tan 50^\circ} = \frac{286 \times \sqrt{3} \times 1.2}{\sqrt{3} - 1.2} \approx 1117$.
故雾灵山主峰的海拔约为 $1117 + 1000 = 2117$ 米.
10. C 因为 $f(x)$ 为偶函数,且在 $[0, +\infty)$ 上单调递减,所以 $f(x)$ 在 $(-\infty, 0]$ 上单调递增.
由 $f(x-1) > f(x)$,得 $|x-1| < |x|$,解得 $x > \frac{1}{2}$,即不等式 $f(x-1) > f(x)$ 的解集为 $(\frac{1}{2}, +\infty)$.
11. A 由 $f(x) = 0$,得 $m - x^2 - 2\ln x$,令 $g(x) = x^2 - 2\ln x$.
则 $g'(x) = 2x - \frac{2}{x} = \frac{2(x-1)(x+1)}{x}$,所以 $g(x)$ 在 $[\frac{1}{e}, 1]$ 上单调递减,在 $[1, e^2]$ 上单调递增.
因为 $g(x)_{\min} = g(1) = 1$, $g(\frac{1}{e}) = \frac{1}{e^2} - 2 < g(e^2) = e^2 - 4$,所以 m 的取值范围为 $(1, \frac{1}{e^2} + 2]$.
12. A 设 $A(x_1, y_1)$, $N(x_1, y_1)$,则 $B(x_1, y_1)$, $M(x_0, y_0)$.
因为 $k_{AN} = \frac{y_1 - y_0}{x_1 - x_0}$, $k_{BN} = \frac{y_1 - y_0}{x_1 - x_0}$,且 $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1$, $\frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1$,所以 $k_{BN} \cdot k_{AN} = \frac{b^2}{a^2} = e^2 - 1$.
因为 $k_{BN} \cdot k_{AN} = -3$,所以 $\frac{k_{AN}}{k_{BN}} = -\frac{1}{3}$.因为 $\overrightarrow{AP} = \frac{3}{2} \overrightarrow{AM}$,所以点 P 的坐标为 $(x_0, -2y_0)$.

因为 $k_{BN} \cdot k_{BP} = -\frac{y_0}{2x_0} = -\frac{1}{2} k_{BN}$,所以 $\frac{k_{BN}}{k_{BP}} = -\frac{e^2 - 1}{3} = -\frac{1}{2}$,解得 $e = \frac{\sqrt{10}}{2}$.

13. -4 作出可行域(图略)可知,当直线 $x + 4y - z = 0$ 经过点 $(4, -2)$ 时, z 最小,且最小值为 -4.

14. $4\sqrt{2}$ 因为圆 C 的圆心为 $(2, -3)$,半径为 4,所以圆心到直线 l 的距离 $d = \frac{|2 - (-3) - 1|}{\sqrt{2}} = 2\sqrt{2}$,故直线 l

被圆 C 截得的弦长为 $2\sqrt{r^2 - d^2} = 4\sqrt{2}$.

15. 820; 3 因为 $\{a_n\}$ 为正项等比数列,所以 $S_2, S_1 - S_2, S_3 - S_1, \dots$ 也成等比数列.

因为 $S_2 = 1, S_3 = 91$,所以 $(S_1 - 1)^2 = 1 \times (91 - S_1)$,得 $S_1^2 - S_1 - 90 = (S_1 - 10)(S_1 + 9) = 0$.因为 $a_n > 0$,所以

所以由(1)可知,点A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z, AA, AB, AC, AD, AE, AF, AG, AH, AI, AJ, AK, AL, AM, AN, AO, AP, AQ, AR, AS, AT, AU, AV, AW, AX, AY, AZ, BA, BB, BC, BD, BE, BF, BG, BH, BI, BJ, BK, BL, BM, BN, BO, BP, BQ, BR, BS, BT, BU, BV, BW, BX, BY, BZ, CA, CB, CC, CD, CE, CF, CG, CH, CI, CJ, CK, CL, CM, CN, CO, CP, CQ, CR, CS, CT, CU, CV, CW, CX, CY, CZ, DA, DB, DC, DD, DE, DF, DG, DH, DI, DJ, DK, DL, DM, DN, DO, DP, DQ, DR, DS, DT, DU, DV, DW, DX, DY, DZ, EA, EB, EC, ED, EE, EF, EG, EH, EI, EJ, EK, EL, EM, EN, EO, EP, EQ, ER, ES, ET, EU, EV, EW, EX, EY, EZ, FA, FB, FC, FD, FE, FF, FG, FH, FI, FJ, FK, FL, FM, FN, FO, FP, FQ, FR, FS, FT, FU, FV, FW, FX, FY, FZ, GA, GB, GC, GD, GE, GF, GG, GH, GI, GJ, GK, GL, GM, GN, GO, GP, GQ, GR, GS, GT, GU, GV, GW, GX, GY, GZ, HA, HB, HC, HD, HE, HF, HG, HH, HI, HJ, HK, HL, HM, HN, HO, HP, HQ, HR, HS, HT, HU, HV, HW, HX, HY, HZ, IA, IB, IC, ID, IE, IF, IG, IH, II, IJ, IK, IL, IM, IN, IO, IP, IQ, IR, IS, IT, IU, IV, IW, IX, IY, IZ, JA, JB, JC, JD, JE, JF, JG, JH, JI, JJ, JK, JL, JM, JN, JO, JP, JQ, JR, JS, JT, JU, JV, JW, JX, JY, JZ, KA, KB, KC, KD, KE, KF, KG, KH, KI, KJ, KK, KL, KM, KN, KO, KP, KQ, KR, KS, KT, KU, KV, KW, KX, KY, KZ, LA, LB, LC, LD, LE, LF, LG, LH, LI, LJ, LK, LL, LM, LN, LO, LP, LQ, LR, LS, LT, LU, LV, LW, LX, LY, LZ, MA, MB, MC, MD, ME, MF, MG, MH, MI, MJ, MK, ML, MM, MN, MO, MP, MQ, MR, MS, MT, MU, MV, MW, MX, MY, MZ, NA, NB, NC, ND, NE, NF, NG, NH, NI, NJ, NK, NL, NM, NN, NO, NP, NQ, NR, NS, NT, NU, NV, NW, NX, NY, NZ, OA, OB, OC, OD, OE, OF, OG, OH, OI, OJ, OK, OL, OM, ON, OO, OP, OQ, OR, OS, OT, OU, OV, OW, OX, OY, OZ, PA, PB, PC, PD, PE, PF, PG, PH, PI, PJ, PK, PL, PM, PN, PO, PP, PQ, PR, PS, PT, PU, PV, PW, PX, PY, PZ, QA, QB, QC, QD, QE, QF, QG, QH, QI, QJ, QK, QL, QM, QN, QO, QP, QQ, QR, QS, QT, QU, QV, QW, QX, QY, QZ, RA, RB, RC, RD, RE, RF, RG, RH, RI, RJ, RK, RL, RM, RN, RO, RP, RQ, RR, RS, RT, RU, RV, RW, RX, RY, RZ, SA, SB, SC, SD, SE, SF, SG, SH, SI, SJ, SK, SL, SM, SN, SO, SP, SQ, SR, SS, ST, SU, SV, SW, SX, SY, SZ, TA, TB, TC, TD, TE, TF, TG, TH, TI, TJ, TK, TL, TM, TN, TO, TP, TQ, TR, TS, TT, TU, TV, TW, TX, TY, TZ, UA, UB, UC, UD, UE, UF, UG, UH, UI, UJ, UK, UL, UM, UN, UO, UP, UQ, UR, US, UT, UU, UV, UW, UX, UY, UZ, VA, VB, VC, VD, VE, VF, VG, VH, VI, VJ, VK, VL, VM, VN, VO, VP, VQ, VR, VS, VT, VU, VV, VW, VX, VY, VZ, WA, WB, WC, WD, WE, WF, WG, WH, WI, WJ, WK, WL, WM, WN, WO, WP, WQ, WR, WS, WT, WU, WV, WW, WX, WY, WZ, XA, XB, XC, XD, XE, XF, XG, XH, XI, XJ, XK, XL, XM, XN, XO, XP, XQ, XR, XS, XT, XU, XV, XW, XX, XY, XZ, YA, YB, YC, YD, YE, YF, YG, YH, YI, YJ, YK, YL, YM, YN, YO, YP, YQ, YR, YS, YT, YU, YV, YW, YX, YY, YZ, ZA, ZB, ZC, ZD, ZE, ZF, ZG, ZH, ZI, ZJ, ZK, ZL, ZM, ZN, ZO, ZP, ZQ, ZR, ZS, ZT, ZU, ZV, ZW, ZX, ZY, ZZ, AA, AB, AC, AD, AE, AF, AG, AH, AI, AJ, AK, AL, AM, AN, AO, AP, AQ, AR, AS, AT, AU, AV, AW, AX, AY, AZ, BA, BB, BC, BD, BE, BF, BG, BH, BI, BJ, BK, BL, BM, BN, BO, BP, BQ, BR, BS, BT, BU, BV, BW, BX, BY, BZ, CA, CB, CC, CD, CE, CF, CG, CH, CI, CJ, CK, CL, CM, CN, CO, CP, CQ, CR, CS, CT, CU, CV, CW, CX, CY, CZ, DA, DB, DC, DD, DE, DF, DG, DH, DI, DJ, DK, DL, DM, DN, DO, DP, DQ, DR, DS, DT, DU, DV, DW, DX, DY, DZ, EA, EB, EC, ED, EE, EF, EG, EH, EI, EJ, EK, EL, EM, EN, EO, EP, EQ, ER, ES, ET, EU, EV, EW, EX, EY, EZ, FA, FB, FC, FD, FE, FF, FG, FH, FI, FJ, FK, FL, FM, FN, FO, FP, FQ, FR, FS, FT, FU, FV, FW, FX, FY, FZ, GA, GB, GC, GD, GE, GF, GG, GH, GI, GJ, GK, GL, GM, GN, GO, GP, GQ, GR, GS, GT, GU, GV, GW, GX, GY, GZ, HA, HB, HC, HD, HE, HF, HG, HH, HI, HJ, HK, HL, HM, HN, HO, HP, HQ, HR, HS, HT, HU, HV, HW, HX, HY, HZ, IA, IB, IC, ID, IE, IF, IG, IH, II, IJ, IK, IL, IM, IN, IO, IP, IQ, IR, IS, IT, IU, IV, IW, IX, IY, IZ, JA, JB, JC, JD, JE, JF, JG, JH, JI, JJ, JK, JL, JM, JN, JO, JP, JQ, JR, JS, JT, JU, JV, JW, JX, JY, JZ, KA, KB, KC, KD, KE, KF, KG, KH, KI, KJ, KK, KL, KM, KN, KO, KP, KQ, KR, KS, KT, KU, KV, KW, KX, KY, KZ, LA, LB, LC, LD, LE, LF, LG, LH, LI, LJ, LK, LL, LM, LN, LO, LP, LQ, LR, LS, LT, LU, LV, LW, LX, LY, LZ, MA, MB, MC, MD, ME, MF, MG, MH, MI, MJ, MK, ML, MM, MN, MO, MP, MQ, MR, MS, MT, MU, MV, MW, MX, MY, MZ, NA, NB, NC, ND, NE, NF, NG, NH, NI, NJ, NK, NL, NM, NN, NO, NP, NQ, NR, NS, NT, NU, NV, NW, NX, NY, NZ, OA, OB, OC, OD, OE, OF, OG, OH, OI, OJ, OK, OL, OM, ON, OO, OP, OQ, OR, OS, OT, OU, OV, OW, OX, OY, OZ, PA, PB, PC, PD, PE, PF, PG, PH, PI, PJ, PK, PL, PM, PN, PO, PP, PQ, PR, PS, PT, PU, PV, PW, PX, PY, PZ, QA, QB, QC, QD, QE, QF, QG, QH, QI, QJ, QK, QL, QM, QN, QO, QP, QQ, QR, QS, QT, QU, QV, QW, QX, QY, QZ, RA, RB, RC, RD, RE, RF, RG, RH, RI, RJ, RK, RL, RM, RN, RO, RP, RQ, RR, RS, RT, RU, RV, RW, RX, RY, RZ, SA, SB, SC, SD, SE, SF, SG, SH, SI, SJ, SK, SL, SM, SN, SO, SP, SQ, SR, SS, ST, SU, SV, SW, SX, SY, SZ, TA, TB, TC, TD, TE, TF, TG, TH, TI, TJ, TK, TL, TM, TN, TO, TP, TQ, TR, TS, TT, TU, TV, TW, TX, TY, TZ, UA, UB, UC, UD, UE, UF, UG, UH, UI, UJ, UK, UL, UM, UN, UO, UP, UQ, UR, US, UT, UU, UV, UW, UX, UY, UZ, VA, VB, VC, VD, VE, VF, VG, VH, VI, VJ, VK, VL, VM, VN, VO, VP, VQ, VR, VS, VT, VU, VV, VW, VX, VY, VZ, WA, WB, WC, WD, WE, WF, WG, WH, WI, WJ, WK, WL, WM, WN, WO, WP, WQ, WR, WS, WT, WU, WV, WW, WX, WY, WZ, XA, XB, XC, XD, XE, XF, XG, XH, XI, XJ, XK, XL, XM, XN, XO, XP, XQ, XR, XS, XT, XU, XV, XW, XX, XY, XZ, YA, YB, YC, YD, YE, YF, YG, YH, YI, YJ, YK, YL, YM, YN, YO, YP, YQ, YR, YS, YT, YU, YV, YW, YX, YY, YZ, ZA, ZB, ZC, ZD, ZE, ZF, ZG, ZH, ZI, ZJ, ZK, ZL, ZM, ZN, ZO, ZP, ZQ, ZR, ZS, ZT, ZU, ZV, ZW, ZX, ZY, ZZ

因为 $|AN| = \sqrt{\left(-\frac{9}{17}\right)^2 + \left(1 - \frac{19}{17}\right)^2} = \frac{\sqrt{85}}{17}$.

所以 $|QA| + |QC|$ 的最大值为 $\frac{\sqrt{85}}{17}$ 12分

21. 解: (1) 因为 $|PF_1| + |PF_2| + |F_1F_2| = 2 - 2\sqrt{2}$, $|F_1F_2| = 2$, 2分

所以 $|PF_1| + |PF_2| = 2a = 2\sqrt{2}$, 3分

所以 $b^2 = a^2 - c^2 = 2 - 1 = 1$, 4分

故椭圆 C 的方程是 $\frac{x^2}{2} + y^2 = 1$ 5分

(2) 由题意可知直线 l 的斜率存在, 设直线 l 的方程为 $y = kx + m$,

联立 $\begin{cases} \frac{x^2}{2} + y^2 = 1, \\ y = kx + m, \end{cases}$ 得 $(1 + 2k^2)x^2 - 4kmx - 2m^2 - 2 = 0$.

设 $A(x_1, y_1), B(x_2, y_2)$,

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则 $\Delta = 16k^2m^2 - 4(1 + 2k^2)(2m^2 - 2) - 16k^2 - 8m^2 + 8 > 0$,

且 $x_1 + x_2 = \frac{4km}{1 + 2k^2}$, $x_1x_2 = \frac{2m^2 - 2}{1 + 2k^2}$ 6分

因为 $kx_1 + y_1 + kx_2 + y_2 = 0$, 所以 $\frac{y_1}{x_1 - 1} + \frac{y_2}{x_2 - 1} = 0$ 8分

所以 $(kx_1 + m)(x_2 - 1) + (kx_2 + m)(x_1 - 1) - 2kx_1x_2 - (k - m)(x_1 + x_2) - 2m = 0$.

所以 $2k \cdot \frac{2m^2 - 2}{1 + 2k^2} + (k - m) \cdot \frac{4km}{1 + 2k^2} - 2m = 0$, 化简得 $m = -2k$, 10分

所以直线 l 的方程为 $y = kx - 2k = k(x - 2)$,

故直线 l 过定点 (2, 0). 12分

22. (1) 解: 因为 $f(x)$ 在 $(0, +\infty)$ 上单调递增, 所以 $f'(x) \geq 0$ 在 $(0, +\infty)$ 上恒成立. 1分

因为 $f'(x) = \frac{\ln x}{x^2} + a$, 所以 $-\frac{\ln x}{x^2} + a \geq 0$, 即 $a \geq \frac{\ln x}{x^2}$ 2分

令 $g(x) = \frac{\ln x}{x^2}$, 则 $g'(x) = \frac{1 - 2\ln x}{x^3}$. 由 $g'(x) = 0$, 得 $x = \sqrt{e}$ 3分

因为 $g(x)$ 在 $(0, \sqrt{e})$ 上单调递增, 在 $(\sqrt{e}, +\infty)$ 上单调递减,

所以 $g(x)_{\max} = g(\sqrt{e}) = \frac{1}{2e}$, 故 $a \geq \frac{1}{2e}$. 即 a 的取值范围是 $[\frac{1}{2e}, +\infty)$ 5分

(2) 证明: 因为 $f'(x) = \frac{\ln x}{x^2} + a$, 所以 $f'(x) = 0$ 的根即直线 $y = a$ 与 $y = \frac{\ln x}{x^2}$ 图像的交点的横坐标.

由(1)知 $y = \frac{\ln x}{x^2}$ 在 $(0, \sqrt{e})$ 上单调递增, 在 $(\sqrt{e}, +\infty)$ 上单调递减. 6分

设 $x_1 < x_2$, 因为 $f'(x) = 0$ 有两个根, 且当 $x > 1$ 时, $\frac{\ln x}{x^2} > 0$,

所以 $x_1 \in (1, \sqrt{e}), x_2 \in (\sqrt{e}, +\infty)$ 7分

令 $F(x) = f'(x) - f'(\frac{c}{x}) = \frac{\ln x}{x^2} - \frac{\ln \frac{c}{x}}{(\frac{c}{x})^2} = \frac{\ln x}{x^2} - \frac{x^2 \ln x}{c^2} - \frac{\ln x}{x^2}, x \in (1, \sqrt{e})$, 8分

则 $F'(x) = \frac{x(1 - 2\ln x)}{x^3} - \frac{1 - 2\ln x}{c^2} = (1 - 2\ln x)(\frac{x}{c^2} - \frac{1}{x^3})$.

因为 $x \in (1, \sqrt{e})$, 所以 $1 - 2\ln x > 0$.

设 $h(x) = \frac{x}{c^2} - \frac{1}{x^3}, x \in (1, \sqrt{e})$.

因为 $h(x)$ 在 $(1, \sqrt{e})$ 上单调递增, 所以 $h(x) < h(\sqrt{e}) = 0$,

所以 $F(x) < 0$, 所以 $F(x)$ 在 $(1, \sqrt{e})$ 上单调递减. 9分

因为 $F(x) > F(\sqrt{e}) = 0$, 所以 $f'(x) > f'(\frac{c}{x})$ 10分

因为 $x_1 \in (1, \sqrt{e})$, 所以 $f'(x_1) > f'(\frac{c}{x_1})$.

因为 $f'(x_1) = f'(x_2)$, 所以 $f'(x_2) > f'(\frac{c}{x_1})$ 11分

因为 $x_2, \frac{c}{x_1} \in (\sqrt{e}, +\infty)$, 且 $f'(x)$ 在 $(\sqrt{e}, +\infty)$ 上单调递增,

所以 $x_2 > \frac{c}{x_1}$, 即 $x_1x_2 > c$ 12分

因为 $q^2 = \frac{S_1 - S_2}{S_2} = 9$, 且 $a_n > 0$, 所以 $q = 3$.

16. $2x - y - 7 = 0$ 设 $A(x_1, y_1), B(x_2, y_2)$, 则 $\begin{cases} x_1^2 - 8y_1 \\ x_2^2 - 8y_2 \end{cases}$ 两式相减得 $x_1^2 - x_2^2 = 8(y_1 - y_2)$, 所以 $\frac{y_1 - y_2}{x_1 - x_2} = \frac{x_1 + x_2}{8}$. 因为线段 AB 的中点坐标为 $(8, 9)$, 所以直线 l 的斜率 $k = \frac{y_1 - y_2}{x_1 - x_2} = \frac{x_1 + x_2}{8} = 2$, 故直线 l 的方程为 $y - 9 = 2(x - 8)$, 即 $2x - y - 7 = 0$.

17. 解: (1) 当 $n=1$ 时, $a_1 - S_1 = \frac{3}{2} + \frac{5}{2} - 4$, 1分

当 $n \geq 2$ 时, $a_n = S_n - S_{n-1} = \frac{3}{2}n^2 + \frac{5}{2}n - [\frac{3}{2}(n-1)^2 + \frac{5}{2}(n-1)] = 3n + 1$, 3分

因为当 $n=1$ 时, $3 \times 1 + 1 = 4$, 所以 $a_n = 3n + 1$ 5分

(2) 因为 $\frac{3}{a_n a_{n+1}} = \frac{3}{(3n+1)(3n+4)} = \frac{1}{3n+1} - \frac{1}{3n+4}$, 8分

所以 $T_n = (\frac{1}{4} - \frac{1}{7}) + (\frac{1}{7} - \frac{1}{10}) + \dots + (\frac{1}{3n+1} - \frac{1}{3n+4})$
 $= \frac{1}{4} - \frac{1}{3n+4} = \frac{3n}{12n+16}$ 10分

18. 解: (1) 因为 $3b \cos C - 5a \sin A - 3c \cos B$, 3分

所以 $3 \sin B \cos C + 3 \sin C \cos B = 3 \sin(B+C) = 5 \sin^2 A$, 4分

即 $3 \sin A = 5 \sin^2 A$, 4分

因为 $\sin A \neq 0$, 所以 $\sin A = \frac{3}{5}$ 6分

(2) 因为 $a < b$, 所以 $\cos A = \sqrt{1 - \sin^2 A} = \frac{4}{5}$ 7分

因为 $a^2 = b^2 + c^2 - 2bc \cos A$, $a = 3, b = 5$, 7分

所以 $9 - 25 - c^2 - 2 \times 5c \times \frac{4}{5} = 0$, 所以 $c^2 + 8c - 16 = 0$, 9分

解得 $c = 4$, 10分

故 $\triangle ABC$ 的面积为 $\frac{1}{2} bc \sin A = \frac{1}{2} \times 5 \times 4 \times \frac{3}{5} = 6$ 12分

19. 解: (1) 设双曲线 C 的方程为 $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 (a > 0, b > 0)$, 则 $\begin{cases} \frac{a}{b} = \frac{\sqrt{6}}{3} \\ c = \sqrt{5} \end{cases}$, 2分

结合 $c^2 = a^2 + b^2$, 得 $\begin{cases} a = \sqrt{2} \\ b = \sqrt{3} \end{cases}$, 4分

故双曲线 C 的标准方程为 $\frac{y^2}{2} - \frac{x^2}{3} = 1$ 5分

(2) 由题意可知, 直线 l 的斜率存在, 设直线 $l: y = kx + 4, A(x_1, y_1), B(x_2, y_2)$.

联立方程组 $\begin{cases} y = kx + 4 \\ \frac{y^2}{2} - \frac{x^2}{3} = 1 \end{cases}$, 消去 y 得 $(3k^2 - 2)x^2 + 24kx + 42 = 0$,

则 $x_1 + x_2 = \frac{24k}{3k^2 - 2}, x_1 x_2 = \frac{42}{3k^2 - 2}$ 7分

所以 $y_1 y_2 = (kx_1 + 4)(kx_2 + 4) = k^2 x_1 x_2 + 4k(x_1 + x_2) + 16 = \frac{42k^2}{3k^2 - 2} + \frac{96k^2}{3k^2 - 2} + 16 = \frac{-6k^2 - 32}{3k^2 - 2}$.

因为原点 O 在以 AB 为直径的圆上, 所以 $OA \perp OB$,

即 $\vec{OA} \cdot \vec{OB} = x_1 x_2 + y_1 y_2 = \frac{42}{3k^2 - 2} + \frac{-6k^2 - 32}{3k^2 - 2} = 0$,

解得 $k = \frac{5}{3}$ 10分

故直线 l 的方程为 $y = \frac{5}{3}x + 4$ 12分

20. 解: (1) 若直线 l 的斜率不存在, 则 l 的方程为 $x = 1$,

此时直线 l 与圆 C 相切, 符合题意. 2分

若直线 l 的斜率存在, 设直线 l 的方程为 $y + 2 = k(x - 1)$, 即 $kx - y - k - 2 = 0$,

因为直线 l 与圆 C 相切, 所以圆心 $(-1, 1)$ 到 l 的距离为 2,

即 $\frac{|2k + 3|}{\sqrt{k^2 + 1}} = 2$, 解得 $k = -\frac{5}{12}$, 4分

所以直线 l 的方程为 $y + 2 = -\frac{5}{12}(x - 1)$, 即 $5x + 12y + 19 = 0$.

综上, 直线 l 的方程为 $x = 1$ 或 $5x + 12y + 19 = 0$ 6分