一、单选

1	2	3	4	5	6	7	8
C	B	C	A	B	D	C	D

二、多选

9	10	11	12
BD	ABC	ACD	BC

三、填空

13、$-\frac{3}{4}$

14、$[-2,2]$

15、$\frac{4}{3}\pi$

16、$\frac{15}{64}$

17 (1) $a_1=2$, $a_2=4$, $a_3=8$.

$q=\frac{a_2}{a_1}=2$.

$\Rightarrow a_n=a_1q^{n-1}=2\cdot 2^{n-1}=2^n$.

(2). $b_n=2^n+(-1)^n\log_2 2^n=2^n+(-1)^n\cdot n$.

$S_n=2^1+(-1)+2^2+2+2^3+(-3)\cdots\cdots+2^n+(-1)^n\cdot n$.

$=2^1+2^2+2^3+2^4\cdots\cdots 2^n+(-1)+2+(-3)\cdots\cdots+(-1)^n\cdot n$

若n为奇数时.

$S_n=\frac{2^1(1-2^n)}{1-2}+(-1+2)+(-3+4)\cdots\cdots+[-(n-2)+(n-1)]-n$.

$=2^{n+1}-2+(\frac{n-1}{2})\times 1-n$.

$=2^{n+1}-\frac{n}{2}-\frac{5}{2}$

若n为偶数时.

$S_n=\frac{2^1(1-2^n)}{1-2}+(-1+2)+(-3+4)\cdots\cdots[-(n-1)+n]$

$=2^{n+1}-2+\frac{n}{2}$.

因此: $S_n=\begin{cases}2^{n+1}-\frac{n}{2}-\frac{5}{2} & (n\text{为奇数})\\ 2^{n+1}+\frac{n}{2}-2 & (n\text{为偶数}).\end{cases}$

18.

(1) $\because S_{\triangle ABC}=(\frac{1}{2}a^2-b^2)\sin C=\frac{1}{2}ab\sin C$

$\therefore \frac{1}{2}a^2-b^2=\frac{1}{2}ab$

$a^2-2b^2=ab$

$\frac{a^2}{b^2}-\frac{a}{b}-2=0$

$(\frac{a}{b}-2)\cdot(\frac{a}{b}+1)=0$

则 $\frac{a}{b}=2$ 或 -1(舍)　　$\therefore a=2b$

即 $\sin A=2\sin B$ 得证.

(2). $\because a\cos C=\frac{3}{2}b$

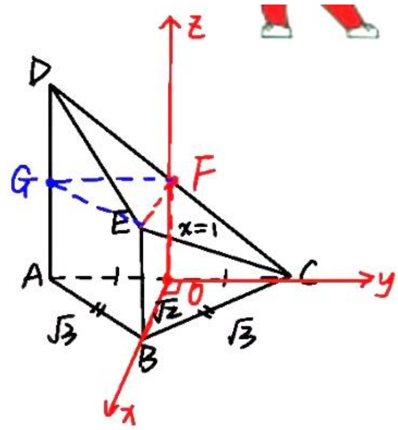
$\therefore \cos C=\frac{3b}{2a}=\frac{a^2+b^2-c^2}{2ab}$

$2b^2=a^2-c^2$

又 $\because a=2b$　　$\therefore c^2=2b^2 \Rightarrow c=\sqrt{2}b$

$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{b^2+2b^2-4b^2}{2\cdot b\cdot\sqrt{2}b}=-\frac{\sqrt{2}}{4}$

19. (1) 证明：



分别取CD、CA中点E、F. 连接EF、FO.

$\because OF \parallel AD$，$AD \perp$ 面ABC

$\therefore OF \perp$ 面ABC

$\because OB \subset$ 面ABC

$\therefore OF \perp OB$

$\because OF \underset{=}{\parallel} \frac{1}{2}AD \underset{=}{\parallel} BE$

$\therefore$ 四边形$OBEF$为平行四边形

$\because AB = BC$

$\therefore OB \perp AC$

$\because AC \cap OF = O$. AC、$OF \subset$ 面ACD

$\therefore OB \perp$ 面ACD

又$\because EF \parallel OB$

$\therefore EF \perp$ 面ACD

又$\because EF \subset$ 面CDE

$\therefore$ 面$CDE \perp$ 面ACD

(2) 如图所示，以O为原点，分别以OB、OC、OF为x、y、z轴，建立空间直角坐标系. 取AD中点G.

设$BE = x$，$\therefore AD = 2BE = x$

$\therefore V = V_{C-OBEF} + V_{D-GEF} + V_{GEF-ABO}$

$\sqrt{2} = \frac{1}{3}\cdot\sqrt{2}x\cdot 1 + \frac{1}{3}\cdot\frac{\sqrt{2}}{2}\cdot x + \frac{\sqrt{2}}{2}\cdot x$

解得 $x = 1$

$\therefore C(0,1,0)$ $E(\sqrt{2},0,1)$ $A(0,-1,0)$

$B(\sqrt{2},0,0)$ $D(0,-1,2)$

$\overrightarrow{CE} = (\sqrt{2},-1,1)$ $\overrightarrow{AB} = (\sqrt{2},1,0)$

$\overrightarrow{BE} = (0,0,1)$

设面ABED的法向量为$\vec{n} = (x,y,z)$

$\therefore \begin{cases} \overrightarrow{AB}\cdot\vec{n} = 0 \\ \overrightarrow{BE}\cdot\vec{n} = 0 \end{cases}$ 即 $\begin{cases} \sqrt{2}x + y = 0 \\ z = 0 \end{cases}$

令 $x = 1$，$y = -\sqrt{2}$

$\therefore \vec{n} = (1,-\sqrt{2},0)$

设CE与面ABED所成角为θ.

$$\therefore \sin\theta = |\cos\langle\overrightarrow{CE},\vec{n}\rangle| = \frac{|\overrightarrow{CE}\cdot\vec{n}|}{|\overrightarrow{CE}|\cdot|\vec{n}|} = \frac{|\sqrt{2}+\sqrt{2}|}{\sqrt{2+1+1}\cdot\sqrt{1+2}} = \frac{\sqrt{6}}{3}$$

20. 解：(1)

x^2	1	4	9	16	25
y	4.9	5.8	6.8	8.3	10.2

$$\overline{x^2}=\frac{x_1^2+x_2^2+x_3^2+x_4^2+x_5^2}{5}=11$$

$$\bar{y}=\frac{y_1+y_2+y_3+y_4+y_5}{5}=7.2$$

$$\sum_{i=1}^{5}(x_i^2-\overline{x^2})(y_i-\bar{y})=(-10)\times(-2.3)+(-7)\times(-1.4)+(-2)\times(-0.4)+5\times1.1+14\times3=81.1$$

$\sum_{i=1}^{5}(x_i^2-\overline{x^2})^2=374$，$\hat{u}=\dfrac{\sum_{i=1}^{5}(x_i^2-\overline{x^2})\cdot(y_i-\bar{y})}{\sum_{i=1}^{5}(x_i^2-\overline{x^2})^2}\approx0.2$，$\hat{v}=\bar{y}-\hat{u}\overline{x^2}=7.2-0.2\times11=5$

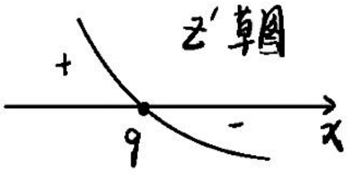
则 $\hat{y}=0.2x^2+5$

(2) $z=24\sqrt{x}-\dfrac{5(0.2x^2+5)+2}{\sqrt{x}}=24\sqrt{x}-\dfrac{x^2+27}{\sqrt{x}}$

$=24x^{\frac{1}{2}}-x^{\frac{3}{2}}-27\cdot x^{-\frac{1}{2}}$

$z'=12x^{-\frac{1}{2}}-\frac{3}{2}x^{\frac{1}{2}}+\frac{27}{2}x^{-\frac{3}{2}}=\frac{12}{\sqrt{x}}-\frac{3}{2}\sqrt{x}+\frac{27}{2}\frac{1}{x^{\frac{3}{2}}}$

易知 z' 随着 x 的递增而递减，且当 $x=9$ 时，$z'=0$



z' 草图

即 z 在 $(0,9)$ 单调增；在 $(9,12)$ 单调减.

当 $x=9$ 时，z 最大

即9月的月投报值最大。

21 题

(1) 设 $M(x,y)$

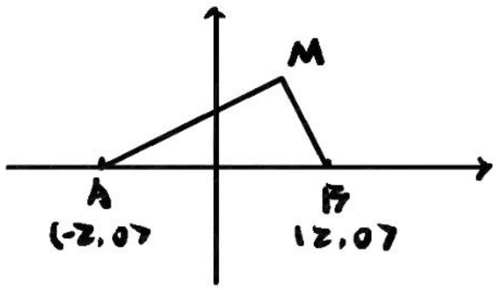
$k_{AM}=\dfrac{y}{x+2}\ (x\neq -2)$ $\quad k_{BM}=\dfrac{y}{x-2}\ (x\neq 2)$

$\therefore k_{AM}\cdot k_{BM}=\dfrac{y^2}{x^2-4}=-\dfrac{3}{4}$

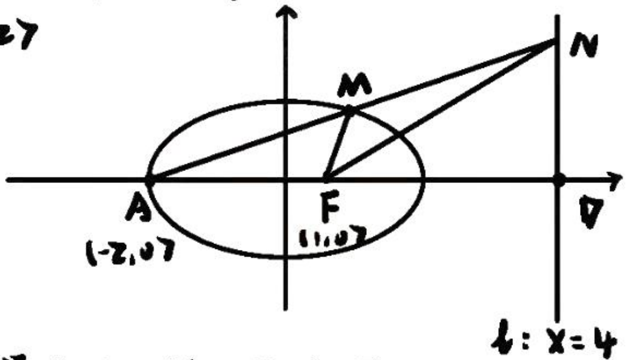
$\therefore 4y^2=12-3x^2$

$\therefore 4y^2+3x^2=12$

$\therefore \dfrac{x^2}{4}+\dfrac{y^2}{3}=1\ (x\neq \pm 2)$



(2)



设 $AM: x=my-2$

联立 $\begin{cases}3x^2+4y^2=12\\ x=my-2\end{cases}$ 得 $3(my-2)^2+4y^2=12$

$\therefore 3(m^2y^2-4my+4)+4y^2=12$

$\therefore (3m^2+4)y^2-12my=0$

$\therefore y_M=\dfrac{12m}{3m^2+4}$

$\therefore x_M=\dfrac{6m^2-8}{3m^2+4}$

$\therefore \tan\angle MFD=\dfrac{y_M}{x_M-1}=\dfrac{12m}{3m^2-12}=\dfrac{4m}{m^2-4}$

令 $4=my-2$

$\therefore y_N=\dfrac{6}{m}$

$\therefore N(4,\dfrac{6}{m})$

$\therefore \tan\angle NFD=\dfrac{\frac{6}{m}}{3}=\dfrac{2}{m}$

$\therefore \tan 2\angle NFD=\dfrac{\frac{4}{m}}{1-\frac{4}{m^2}}=\dfrac{\frac{4}{m}}{\frac{m^2-4}{m^2}}=\dfrac{4m}{m^2-4}$

$\therefore 2\angle NFD=\angle MFD$

$\therefore \lambda=2$

22 (1). $f(x)=e^x+\sin x-\cos x$

$f'(x)=e^x+\cos x+\sin x$

$=e^x+\sqrt{2}\sin(x+\frac{\pi}{4})$

①当 $x\in[0,\frac{\pi}{2}]$，$x+\frac{\pi}{4}\in[\frac{\pi}{4},\frac{3\pi}{4}]$，$\sin(x+\frac{\pi}{4})\geq\frac{\sqrt{2}}{2}$，$e^x\geq1$

$\therefore f'(x)\geq1+\sqrt{2}\times\frac{\sqrt{2}}{2}=2$

②. $x\in[\frac{\pi}{2},+\infty)$ 时 $f'(x)\geq e^{\frac{\pi}{2}}-\sqrt{2}>e^{\frac{3}{2}}-\sqrt{2}>e\sqrt{e}-\sqrt{2}>(e-1)\sqrt{2}>\sqrt{2}\cdot\sqrt{2}=2$

$\therefore f'(x)>2$

由①②知 $f'(x)\geq2$

(2). $g(x)=f(x)-2x-1$，$g'(x)=f'(x)-2$

①由(1)知，$x\in[0,+\infty)$ 时，$f'(x)\geq2$，$g'(x)\geq0$

$\therefore g(x)$ 在 $[0,+\infty)$ 是增函数

$\because g(0)=f(0)-1=-1<0$，$g(\frac{\pi}{2})=e^{\frac{\pi}{2}}-\pi>0$

$\therefore g(x)$ 在 $[0,\frac{\pi}{2}]$ 存在唯一零点，在 $(\frac{\pi}{2},+\infty)$ 无零点，

② $x\in(-\infty,-1]$ 时，$g'(x)=e^x+\sqrt{2}\sin(x+\frac{\pi}{4})-2\leq\frac{1}{e}+\sqrt{2}-2<0$

$x\in(-1,0)$ 时，$g'(x)=e^x+\sqrt{2}\sin(x+\frac{\pi}{4})-2<1+1-2=0$

$\therefore g(x)$ 在 $(-\infty,0)$ 是减函数

$g(-\frac{\pi}{2})=e^{-\frac{\pi}{2}}-1+\pi-1>\pi-3>0$

$\therefore g(x)$ 在 $(-\frac{\pi}{2},0)$ 存在唯一零点，在 $(-\infty,-\frac{\pi}{2}]$ 无零点，

综上可知，$g(x)$ 在 R 上有且仅有两个零点，

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